Demand for Collateral, Foreign Holdings of U.S. Treasuries and Taxes on Capital Flows*

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Abstract

This paper proposes a potential new explanation to foreign demand for U.S. Treasuries: a demand for collateral. We solve a model that highlights the mechanism and allows to disentangle the demand for collateral from the demand for safe assets. Countries that buy U.S. Treasuries to store value differ from countries buying U.S. Treasuries for their collateral properties by how active are their credit relations with other countries via repo markets. We use the model to study optimal taxes on international borrowing and capital flows. We start by showing that to achieve full risk sharing demands either a subsidy on repo borrowings that is inversely proportional to the collateral margin and to the yield of U.S. treasuries. Or a subsidy on the price of U.S. treasuries.

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1 Introduction

Foreign holdings of U.S. Treasuries are large and display ample heterogeneity across countries. Table 1 illustrates these facts using an indicator of the relative size of foreign holdings (the average holdings over 2004-2009 of long term U.S. Treasuries and agency securities over the exports plus imports of the country) for a sample of 46 countries (excluding financial hubs). The different statistics are quite similar across income groups.

Table 1: Ratio of holdings of LT Treasuries over exports plus imports

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<th>Average over 2004-2009</th>
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<tr>
<td></td>
<td>Mean</td>
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<td>High Income countries</td>
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<td>Middle Income countries</td>
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<td>Low Income countries</td>
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Source: TIC System. Sample includes the 46 countries with largest holdings excluding financial hubs. Treasuries are defined as both U.S. treasuries and Agency securities. Sample includes both private and official holdings. Income groups follow the World Bank classification. Long Term Treasuries are defined as those with maturity over 1 year.

Understanding the drivers of the foreign demand for U.S. Treasuries is important for both the U.S. and the holding countries. For the U.S. because foreign inflows seem to have lowered interest rates (Warnock and Warnock 2009, Favilukis et al. 2011) and helped to finance U.S. fiscal and current account deficits (Greenspan 2005, Bernanke 2005). For the holding countries because if their accumulation of U.S. assets is due to domestic distortions then identifying them is a prerequisite for successful policies. This is the case of developing economies with ongoing financial reforms (Kawai and Prasad 2011).

This paper proposes a potential new explanation to foreign demand for U.S. Treasuries: a demand for collateral. We solve a model that highlights the mechanism and allows to disentangle

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1Official public holdings account for most of the holdings but private holdings are not trivial. For example, they account for 25% of total holdings in 2010.
the demand for collateral from the demand for safe assets. Then we use the model to study optimal taxes on international borrowing and capital flows.

In the model there are two countries with heterogenous endowments, high in one period, low the next one, as in Woodford (1990). The countries have an incentive to borrow and lend between them to smooth consumption. However, all borrowing and lending must be via repo markets and only U.S. Treasuries can serve as collateral. We have in mind emerging economies that lack assets with good collateral properties for international borrowing. Thus, the collateral margin captures a precise notion of financial underdevelopment, when it is high (low \( m \) in the model) the country lacks domestic collateral internationally accepted. To simplify we assume that U.S. prices are exogenous.

The model has two types of equilibria. In the first one the margin requirement is so severe that there is no demand for collateral. The demand for U.S. assets is driven by a demand for storage of value à la Caballero et al. (2008). Countries buy Treasuries to save for future periods because they do not have any other way to store value. The second type of equilibria is the one that interests us. The margin requirement is loose enough \( (m > 1) \) and the foreign economies buy U.S. assets to do repo lending between them, even if they never achieve full risk-sharing. The demand for U.S. Treasuries is positive even if Treasuries do not pay dividends and have no use as storage of value. This demand may be high for two reasons: 1) High benefits from borrowing (in this model because of low intertemporal elasticity of substitution and/or high heterogeneity in endowments, but the same insights hold in a model where borrowings are driven by investment opportunities instead of by a desire to smooth consumption). Or 2) high margin requirements. These two reasons may be behind Table 1. Some rich countries demand U.S. Treasuries for reason 1, because their repo needs exceed the amount of good collateral they can generate. Medium and low income countries for reason 1, but also for reason 2.

The result that the value of the margin requirement differentiates those two types of equilibria is useful to distinguish between countries that buy U.S. Treasuries to store value, from countries buying internationally acceptable collateral. The first type of countries do not have much interaction with financial markets. When they have high endowments they just save part of them in the U.S. However, countries with demand for collateral also have active repo markets relations with other countries. In practice this involves an active banking system that participates in wholesale money markets. Korea may be a good example, it is a large holder of U.S. Treasuries, and Shin (2010) reports that bank lending far outstrips funding available from retail deposits, banks fill the gap with short-term wholesale funding, especially from international capital markets. Unfortunately we cannot know which fraction of the borrowing is done via repo markets. As we discuss in Section 2 this is an important limitation for applied work.
We know that repo markets are large but we have yet few data about them.

The collateral margin is a friction that reduces welfare. It prevents the non-U.S. countries from achieving risk sharing by depressing credit between foreigners. Optimal financial reform should target it, for example by improving foreigners’ ability to generate good collateral. However, in practice it is very hard to do this. It usually involves structural reforms as reducing corruption or reforming the legal system. It may take many years for a country to affect its collateral margin. The empirical evidence shows that countries can impose taxes and subsidies at a much faster rate. And international institutions as the IMF help in coordinating them, as shown during the recent debate on capital controls (Ostry et al. 2010). Thus, in Section 6 we evaluate two types of short term policies: a subsidy on repo borrowings and another on purchases of U.S. Treasuries. We show that full risk-sharing demands either a subsidy on repo borrowings that is inversely proportional to the collateral margin and to the yield of U.S. Treasuries. Or a subsidy on the price of U.S. Treasuries. In this version of the paper we looked only whether we have full risk sharing but not at which level we have it. The next step will be to study the optimal subsidies.

The paper is related to a number of strands of literature:

1) Methodologically the paper is related to the literature on which borrowing constraints confer assets a collateral value (Bernanke and Gertler 1989; Detemple and Murthy 1997; Geanakoplos 1997; Kiyotaki and Moore 1997; Caballero and Krishnamurthy 2001; Lustig and Van Nieuwerburgh 2005; Coen-Pirani 2005; Fostel and Geanakoplos 2008 or Garleanu and Pedersen 2011 among others). And to recent models with heterogenous agents as Angeletos and Panousi (2011), Buera and Moll (2011) or Cao (2011).

2) By the topics studied the paper is related to three literatures:

2.1) The literature on foreign holdings of U.S. assets has proposed several reasons why foreigners hold U.S. assets. Explanations for official public holdings usually focus on precautionary and mercantilist motives (Dominguez 2010 provides a summary). Forbes (2010) surveys the literature on private-sector holdings and evaluates different theories. She shows that despite strong theoretical support, diversification motives appear to have little impact on patterns of foreign investment. Trade with the U.S. and lack of capital controls explain holdings. But, her strongest and most consistent result is that a country’s level of financial development (measured by the size of the country’s stock and bond markets) is an important factor affecting its share of investment in both U.S. equity and debt markets. Caballero et al. (2008) and Mendoza et al. (2009) provide the existing theoretical link between financial underdevelopment and foreign holdings of U.S. Treasuries. In Caballero et al. (2008) the demand for U.S. assets is driven by
a shortage of "safe" assets in emerging economies. In Mendoza et al. (2009) because U.S. assets provide insurance and risk sharing. Our paper provides an alternative and complementary explanation.

2.2) Several studies have documented that institutional differences help explain differences on international capital flows. For example, Gelos and Wei (2005) show this for both government and corporate transparency; Ferreira and Matos (2008) for disclosure standards and Leuz et al. (2009) for outsider protection. Our paper provides theoretical support for institutional differences affecting the supply of assets with collateral properties.

2.3) Our policy exercises complement the recent literature on capital controls (Korinek 2011 is a recent survey). This literature focuses on pecuniary externalities via borrowers who do not internalize the effects of their borrowing on asset prices. The policy recommendations are taxes on international borrowing or on capital flows. Our model shows a different mechanism implying the opposite policy recommendation, the capital flows alleviate the inefficiency arising from the shortages of domestic collateral.

The rest of the paper is organized as follows. Section 2 motivates the model. Section 3 describes it. Section 4 characterizes its equilibria. Section 5 studies comparative statics. Section 6 analyzes different short term policy options. Section 7 concludes.

2 Motivation

Our paper builds on four assumptions that seem well supported by data:

1) Collateralized lending via repo markets is an important source of funds for financial institutions and large firms. We need to rely on private estimates because there are no official statistics on the overall size of the repo market. For example, Gorton and Metrick (2011) estimate U.S. repo markets to be about the same size, or larger, than the U.S. banking system of $10 trillion. Hördahl and King (2008) estimate gross amounts outstanding at year-end 2007 of roughly $10 trillion in each of the U.S. and Euro markets, and another $1 trillion in the UK repo market. The International Capital Market Association (ICMA) estimates that the size of European repo markets hit a peak of EUR 6,775 billion in June 2007, then dropped to a low in December 2008 of EUR 4,633 billion, and have increased again to EUR 5,908 billion in December 2010 (ICMA 2010, BIS 2010). Moreover, Fitch Ratings (2011) documents that the recent financial crisis has pushed banks to favor secured funding relative to unsecured debt because: 1) the cost differential between secured and unsecured bank debt widened after
2008; 2) the inability of weaker banks to access senior unsecured debt markets. Supporting the increasing importance of repo markets, Lam and Zhang (2010) claim that in China, interbank repo transactions have become the most important form of interbank borrowing due to the problem of non-performing uncollateralized interbank loans.

2) Cross-border lending is important in many emerging economies. For example, Takáts (2010) reports that by the end of 2008, total bank lending of foreign banks and their affiliates exceeded US$ 1,500 billion in emerging Asia, US$ 900 billion in emerging Europe and US$ 800 billion in Latin America. These flows are specially important for countries as Korea, whose banking system has low deposits to loan ratios and needs to borrow in international wholesale funding markets (Shin 2010). The BIS consolidated banking statistics for 2008 show that foreign lending to the Korean banking sector exceeded 40% of Korean’s GDP. Unfortunately we do not know which fraction of the lending was done via repo markets. But we have anecdotal evidence showing the existence of repo lending in Asia collateralized by U.S. Treasuries (Lee 2009).

3) Emerging economies lack assets with good collateral properties (Turner 2002). Ceteris paribus, risk averse lenders prefer collateral whose value is stable and easy to sell (small transaction costs). Developing countries are very volatile (Aguiar and Gopinath 2007) and have legal systems that make very costly to repossess the collateral and defend creditors rights (World Bank 2010).

4) U.S. Treasuries are the assets most widely accepted around the world as collateral. A few examples: around 85% of the participants in the OTC derivatives industry accept them (ISDA 2000). The Korea Securities Depository accepts them as collateral for securities lending and borrowing transactions. In European Repo markets around 25% of the transactions use U.S. assets as collateral (ICMA 2010). Moreover, Bartolini et al. (2010) and Krishnamurthy and Vissing-Jorgensen (2010) provide evidence that Treasuries command a collateral premium.

3 Model

We assume a deterministic economy composed by two agents heterogenous in their endowments. Agent 1 receives endowment \( e \) in odd periods and \( e' \) in even periods. Agent 2 vice versa.\(^2\) They can buy (but not sell) a U.S. Treasury that pays off one unit of final good next period and costs today \( \frac{1}{R_t} \) units. We denote by \( k_i \) the foreign holdings of U.S. Treasuries by the agent \( i \).

\(^2\)In the Appendix, we present a stochastic model with the same insights that we obtain with the deterministic.
The agents can borrow or lend among them at repo rate \( R_t \), but they are subject to a collateral constraint with an exogenous collateral margin \( (m) \). Their borrowings today \( (b_{i,t+1} < 0) \) need to be collateralized by holdings of U.S. assets. We think on this as cross-country repo transactions between countries without collateralizable assets.

Each agent chooses \( \{c_{i,t}, k_{i,t+1}, b_{i,t+1}\} \) to solve

\[
\max_{\{c_{i,t},k_{i,t+1},b_{i,t+1}\}} \sum_{t=0}^{\infty} \beta^t u(c_{i,t}) \tag{1}
\]

subject to her borrowing constraint

\[
c_{i,t} + \frac{1}{R_{US}^t} k_{i,t+1} + \frac{1}{R_t} b_{i,t+1} \leq e_{i,t} + b_{i,t} + k_{i,t} \tag{2}
\]

to the no-short selling constraint

\[
k_{i,t+1} \geq 0 \tag{3}
\]

and to the collateral constraint

\[
b_{i,t+1} \geq -mk_{i,t+1} \tag{4}
\]

**Definition 1** For exogenous \( \{R_{US}^t, e_{i,t}\} \) a competitive equilibrium is a sequence of Repo interest rates \( \{R_t\} \) and allocations \( \{b_{i,t+1}, c_{i,t}, k_{i,t}\} \) such that both agents maximize and the Repo Market clears

\[
b_{1,t+1} + b_{2,t+1} = 0 \tag{5}
\]

4 Equilibria

In this section we characterize the equilibria of the model as a function of \( m \). Basically, for \( m < 1 \) the equilibria display demand for U.S. assets to store wealth, while for \( m > 1 \) the demand for U.S. assets is purely driven by a demand for collateral.\(^3\) We can start by analyzing the problem of the agents in the economy. We will refer to agents with high endowment \((\bar{\tau})\) as the rich, the others as the poor.

\(^3\)If we introduce uncertainty there might be demand for collateral even when over-collateralization, i.e. \( m < 1 \). But the main insights from the deterministic model go through.
4.1 Case 1: \( m=0 \)

When \( m = 0 \) then \( b_{i,t+1} \geq 0 \) and no borrowings are possible. Moreover, given that the Repo market is in zero net supply, equation (5) implies \( b_{i,t+1} = 0 \) at any date. Rich households have only one way to smooth consumption: via the U.S. asset.

We conjecture the following equilibrium: \( c_{it} = \bar{c} \) when an agent is rich and \( c_{it} = \underline{c} \) when she is poor

\[
\bar{c} = \bar{c} - \frac{k}{R^{US}} \quad (6)
\]
\[
\underline{c} = \underline{c} + k. \quad (7)
\]

That is, when the agent is rich she saves by buying \( k_{it} = k \) units of U.S. assets. When she is poor she consumes all her endowment plus the savings stored in U.S. Treasuries. She does not buy any additional U.S. asset, that is, for the poor \( k_{it} = 0 \).

**Proposition 1** Suppose

\[
\beta R^{US} \leq 1 \leq \beta R^{US} u'(\bar{c}) \quad (8)
\]

then there exists an equilibrium under the form (6)-(7).

**Proof.** We need to verify the optimality of the first-order conditions on the asset holding of the consumers. For the rich agent

\[
\frac{1}{R^{US}} u' \left( \bar{c} - \frac{k}{R^{US}} \right) = \beta u' \left( \bar{c} + k \right) \quad (9)
\]

and we can define

\[
\Delta (k) = 1 - \beta R^{US} \frac{u'(\bar{c} + k)}{u' \left( \bar{c} - \frac{k}{R^{US}} \right)}. \]

Condition (8), plus preferences satisfying Inada conditions, imply that \( \Delta (0) \leq 0 \) and

\[
\Delta \left( \frac{\bar{c} - \underline{c}}{1 + \frac{k}{R^{US}}} \right) \geq 0. \quad (8)
\]

Thus, there exists \( k^* \in \left( 0, \frac{\bar{c} - \underline{c}}{1 + \frac{k}{R^{US}}} \right) \) such that \( \Delta (k^*) = 0 \), that is the Euler condition for the rich agent is satisfied.

For the poor agent the non-short selling for U.S. asset is binding:

\[
\frac{1}{R^{US}} u'(\bar{c} + k) \geq \beta u' \left( \bar{c} - \frac{k}{R^{US}} \right). \]
The Euler condition for the poor agent is also satisfied because $\beta R^{US}_t \leq 1$. 

Summarizing, for $m = 0$ the equilibrium displays demand for U.S. assets to store wealth. This is basically the story of Caballero et al. (2008). Emerging economies do not have safe assets to store wealth. Thus, they place their savings in the U.S.

### 4.2 Case 2: $m > 0$

Now we go back to the general case with $m > 0$ that potentially allows for demand for collateral. We will show that for $m \geq 1$ there will be demand for U.S. Treasuries as collateral. However, for $m < 1$, the U.S. asset has no collateral value but only storage value.

Let $\lambda_{i,t}$ denote the Lagrange multiplier on the collateral constraint (4) and $\mu_{i,t}$ denote the Lagrange multiplier on the no-short selling constraint (3). The first-order condition with respect to $k_{i,t+1}$ implies

$$-\frac{1}{R^{US}_t} u'(c_{i,t}) + \beta u'(c_{i,t+1}) + \mu_{i,t} + m\lambda_{i,t} = 0. \quad (10)$$

The first order condition with respect to $b_{i,t+1}$ implies

$$-\frac{1}{R_t} u'(c_{i,t}) + \beta u'(c_{i,t+1}) + \lambda_{i,t} = 0. \quad (11)$$

And the complementarity-slackness conditions are

$$\lambda_{i,t} (b_{i,t+1} + mk_{i,t+1}) = 0$$

$$\mu_{i,t} k_{i,t+1} = 0$$

Given the general formulation, we can prove the following property of equilibria:

**Proposition 2** If $m > 1$ then in equilibrium there is an active repo market with $R_t > R^{US}_t$. If $m \leq 1$ then for any equilibrium, we can find an equivalent equilibrium, in terms of consumption allocation, such that $b_{i,t+1} = 0$ for all $i$. 

Proof. If $m > 1$, (11) and (10) imply

$$
\frac{1}{R_t^{US}} u'(c_{i,t}) = \beta u'(c_{i,t+1}) + \mu_{i,t} + m \lambda_{i,t}
$$

$$
\geq \beta u'(c_{i,t+1}) + \lambda_{i,t}
$$

$$
\geq \frac{1}{R_t} u'(c_{i,t})
$$

so $R_t \geq R_t^{US}$ with strict inequality if $\lambda_{i,t} > 0$.

If $m \leq 1$: Suppose, by contradiction, that there is an active repo market in equilibrium. Then there must be $i$ such that $b_{i,t} > 0$. So $\lambda_{i,t} = 0$. (11) and (10) imply

$$
\frac{1}{R_t^{US}} u'(c_{i,t}) = \beta u'(c_{i,t+1}) + \mu_{i,t}
$$

$$
\geq \beta u'(c_{i,t+1}) = \frac{1}{R_t} u'(c_{i,t})
$$

so $R_t \geq R_t^{US}$. Now, there must be an $i$ such that $b_{i,t} < 0$. The collateral constraint (4) implies that $k_{i,t+1} > 0$ or $\mu_{i,t} = 0$. So

$$
\frac{1}{R_t^{US}} u'(c_{i,t}) = \beta u'(c_{i,t+1}) + m \lambda_{i,t}
$$

$$
\leq \beta u'(c_{i,t+1}) + \lambda_{i,t}
$$

$$
\leq \frac{1}{R_t} u'(c_{i,t})
$$

or $R_t \leq R_t^{US}$. So the two inequalities imply $R_t = R_t^{US}$. Consequently $\lambda_{i,t} = 0$ for the agents who are borrowing, i.e. their collateral constraints are not binding. Now for agents with $b_{i,t+1} > 0$ we can replace their repo lending by holding U.S. Treasuries. For agents with $b_{i,t} < 0$, we can replace $k_{i,t+1}$ by $k_{i,t+1} + b_{i,t+1}$ ($>0$ due to the collateral constraint) and reduce $b_{i,t+1}$ to zero. This gives us an equivalent allocation with no repo lending.

This proposition shows that for $m < 1$ the model is equivalent to the case with $m = 0$ above. When $m > 1$ we will show by construction the existence of an equilibrium with the following form:

1) Rich agents does not buy U.S. assets and lend in the repo market.

2) Poor agents buy U.S. asset and borrow using these assets as collateral

We will look for a stationary equilibrium in which $(c_{it}, k_{i,t+1}, b_{i,t+1}) = (\tau, k, \bar{b})$ if agent $i$ is rich (high endowment) at time $t$, and $(c, k, \bar{b})$ if agent $i$ is poor (low endowment) at time $t$. 10
Lemma 3 In a stationary equilibrium of the form described above for \( m > 1 \), the rate of return \( R \) is an increasing function in \( m \). Moreover, the non-U.S. economies cannot achieve full risk-sharing, i.e., \( \underline{c} \neq \bar{c} \).

**Proof.** Given that the rich agents are not borrowing in the repo market, their collateral constraint are not binding. The first-order condition (11) implies

\[
\frac{1}{R} u'(\underline{c}) = \beta u'(\bar{c}) \tag{12}
\]

Now, the poor agents borrow in the repo markets. Because they have to buy the U.S. asset to use as collateral, the no short selling constraint (3) is not binding, that is \( \mu_{i,t} = 0 \) in (10). The first order condition (11) and (10) become

\[
-\frac{1}{R} u'(\underline{c}) + \beta u'(\bar{c}) + \lambda = 0 \tag{13}
\]

and

\[
-\frac{1}{R^{US}} u'(\underline{c}) + \beta u'(\bar{c}) + m\lambda = 0. \tag{14}
\]

Multiplying the first equation by \( m \) and subtracting it from the second equation imply

\[
-\left(\frac{m}{R} - \frac{1}{R^{US}}\right) u'(\underline{c}) + \beta (m-1) u'(\bar{c}) = 0.
\]

Combining this equation with equation (12) implies

\[
\frac{1}{R} \left(\frac{m}{R} - \frac{1}{R^{US}}\right) = \beta^2 (m-1). \tag{15}
\]

This a quadratic equation in \( R \) and gives a unique solution for \( R \) as a function of \( m \) and \( R^{US} \)

\[
R = mR^{US} \frac{2}{\sqrt{1 + 4m (m-1) \beta^2 (R^{US})^2 + 1}}. \tag{16}
\]

If \( \underline{c} = \bar{c} \) then equation (12) implies \( R = \frac{1}{\beta} \). This result and equation (15) imply \( R^{US} = \frac{1}{\beta} \). However, \( R = R^{US} \) contradicts (13) and (14) for \( m > 1 \). \( \blacksquare \)

The size of the repo market is \( b = mk \). In order to fully determine allocation and prices in
equilibrium, we use the budget constraints of the consumers and the market clearing condition:

\[ c = e + \frac{1}{R} mk - \frac{1}{RUS} k + mk \] (17)

\[ \bar{c} = \bar{e} - \frac{1}{R} mk - mk + k \] (18)

where the last term in (18) represents the dividends received from the holdings of U.S. assets.

Using the expression for \( R \) in (16) and the first order for rich agents (12) implies an equation that determines \( k \), as a result, \( c \) and \( \bar{c} \). For example, under iso-elastic utility function \( u'(c) = c^{-\sigma} \) equation (12) becomes

\[ \frac{\bar{c}}{c} = (\beta R)^{1/\sigma} \]

and using (17) and (18)

\[ \frac{\bar{c} - \frac{1}{R} mk - mk + k}{\bar{e} + \frac{1}{R} mk - \frac{1}{RUS} k + mk} = (\beta R)^{1/\sigma}. \]

So

\[ \bar{c} - \frac{1}{R} mk - mk + k = (\beta R)^{1/\sigma} \left( \bar{e} + \frac{1}{R} mk - \frac{1}{RUS} k + mk \right), \]

or

\[ k = \frac{\bar{c} - (\beta R)^{1/\sigma} \bar{e}}{(\beta R)^{1/\sigma} \left( \frac{m}{R} \frac{1}{RUS} + m \right) + \frac{m}{R} + m - 1}. \] (19)

Thus, in this type of equilibria demand for U.S. assets is completely driven by demand for collateral. And as it shown by equation (19), this demand is positive even if the U.S. asset pays zero dividends and has no storage value.

5 Comparative statics

In this section, we use numerical simulations to explore the qualitative implications of the model. The model is too stylized for a full quantitative analysis. The discount factor \( \beta \) is set to 0.95. For preferences we assume the standard CRRA

\[ u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma} \]

with coefficient of risk aversion \( \sigma \) equal to 3.

Concerning the exogenous dividends and prices of the U.S. assets we set dividends to be 1, and the yield on U.S. asset \( R^{US} \) to be \( 1.1 \times \beta \). Finally, for the endowment parameters we set
$e_L = 10$ and $e_H = 100$. We let $m$ to vary from 0 to 10 to show the different equilibria as a function of $m$. We report the results in Figures 1 to 3.

Figure 1 shows the foreign holdings of U.S. Treasuries as a function of the collateral requirement. The model generates the same maximum demand for two very different reasons. For equilibria with $m \leq 1$ the demand for Treasuries is driven by a demand to store value. For $m > 1$ it is driven by demand for collateral. As $m \to \infty$ the collateral friction is weaker and demand for U.S. assets vanishes. Moreover, markets are more complete as $m \to \infty$, thus the consumption of foreign countries converges to the efficient perfect insurance case. This is illustrated in Figure 2 which shows the consumption of the rich and poor agents as a function of the collateral requirement. Figure 3 shows that the interest rate on repo transactions between foreigners depend on the collateral margin. Rates increase as borrowers need to buy less U.S. collateral.

The previous figures suggest how to disentangle holdings of Treasuries driven by a demand for safe assets from a demand for collateral. We should look for the correlations between foreigners’ repo transactions and foreign holdings of U.S. Treasuries. We illustrate this result in Figure 4. It shows two different equilibria, one with $m = 2$ in which there is only demand for collateral, and another with $m = 0.5$, in which there is only demand for storage. The x-axis
Figure 2: Consumption of foreign economies

Figure 3: Interest rate repo markets
captures the desire for borrowing, the larger is \((e_H - e_H)\) the more uneven consumption and higher the demand to smooth. Countries that demand U.S. assets just to store value \((m \leq 1)\) should have depressed financial relations with other countries. On the other way, when the demand is for collateral reasons, it comes together with borrowings in repo markets.

6 Policy Proposals

In the previous setup the competitive equilibrium is inefficient, the countries cannot achieve perfect risk sharing. The best way to improve would be by affecting \(m\), that is the source of inefficiency. But in practice this is very hard to do, improving \(m\) usually involves structural reforms that take many years to implement. However, countries are able to impose taxes and subsidies at a much faster rate. In this section we explore how a subsidy on repo borrowings or on purchases of U.S. Treasuries can achieve perfect risk sharing. The reason we focus on perfect risk-sharing is because we have heterogenous agents, thus to specify a social welfare function we should weight arbitrarily the different agents. Moreover, despite the collateral constraint, the equilibria are constraint efficient because we do not have the pecuniary externality. That is, in our model the price of the collateral asset (U.S. Treasury) is exogenous as opposed to
6.1 Tax/subsidize the repo borrowing

The setup is the same of Section 3 but now the Government can use lump sum taxes (subsidies) \((T_{i,t})\) to finance a tax (subsidy) on repo borrowing \((\tau)\). Thus, now the agents maximize preferences (1) subject to

\[
c_{i,t} + \frac{1}{R_t^{US}} k_{i,t+1} + \frac{1}{R_t} b_{i,t+1} \leq c_{1,t} + b_{i,t} + k_{i,t} - T_{i,t} \text{ if } b_{i,t+1} \geq 0
\]

\[
c_{i,t} + \frac{1}{R_t^{US}} k_{i,t+1} + \frac{1}{R_t} b_{i,t+1} \leq c_{1,t} + b_{i,t} + k_{i,t} - T_{i,t} \text{ if } b_{i,t+1} < 0,
\]

to the no-short selling constraint (3), and to the collateral constraint (4).

**Definition 2** For exogenous \(\{R_t^{US}, e_{i,t}, T_{i,t}\}\) a competitive equilibrium is a sequence of Repo interest rates \(\{R_t\}\) and allocations \(\{b_{i,t+1}, c_{i,t}, k_{i,t}\}\) such that both agents maximize, the Repo Market clears and transfers are self-financing

\[
\sum_i \left( T_{i,t} + \frac{\tau}{R_t} b_{i,t+1} 1_{\{b_{i,t}<0\}} \right) = 0
\]

(20)

We can show that there is a system of taxes and transfers that can achieve full risk-sharing among foreigners. The borrowers should receive a subsidy in the repo rates they have to pay back. This subsidy should be inversely proportional to \(m\) and the yield of U.S. Treasuries.

**Proposition 4** Let \(\tau^* = \frac{1}{m} \left( \frac{1}{\beta R^{US}} - 1 \right)\), then the foreign economies achieve full risk-sharing.

**Proof.** Similar to Subsection (4.2), the system that determines interest rate and allocations is the bond Euler equation for the rich

\[
\frac{1}{R} u'(\bar{c}) = \beta u'(\bar{c})
\]
and those of the poor

\[-\frac{1 + \tau}{R} u'(c) + \beta u'(c) + \lambda = 0 \]  

(21)

\[-\frac{1}{R^{US}} u'(c) + \beta u'(c) + m\lambda = 0 \]  

(22)

Combining the last two equations yields

\[-\left(\frac{m (1 + \tau)}{R} - \frac{1}{R^{US}}\right) u'(c) + \beta (m - 1) u'(c) = 0.\]

Now in order to obtain full risk sharing ($\bar{c} = c$) we need

\[R = \frac{1}{\beta}\]

\[\left(\frac{m (1 + \tau)}{R} - \frac{1}{R^{US}}\right) = \beta (m - 1)\]

The two equations yield $\tau^*$.4

6.2 Tax/subsidize the foreign holdings of U.S. Treasuries

The setup is the same of Section 3 but now the Government can use lump sum taxes (subsidies) ($T_{i,t}$) to finance a tax (subsidy) ($\tau$) on capital flows with the U.S. Thus, now the agents maximize preferences (1) subject to

\[c_{i,t}^F + \frac{1 - \tau}{R_{i,t}^{US}} k_{i,t+1} + \frac{b_{i,t+1}}{R_t} \leq c_{i,t} + b_{i,t} + k_{i,t} - T_{i,t} \text{ if } b_{i,t+1} \geq 0\]

\[c_{i,t}^F + \frac{1 - \tau}{R_{i,t}^{US}} k_{i,t+1} + \frac{b_{i,t+1}}{R_t} \leq c_{i,t} + b_{i,t} + k_{i,t} - T_{i,t} \text{ if } b_{i,t+1} < 0,\]

to the no-short selling constraint (3), and to the collateral constraint (4).

**Definition 3** For exogenous $\{R_{i,t}^{US}, e_{i,t}, T_{i,t}\}$ a competitive equilibrium is a sequence of Repo interest rates $\{R_t\}$ and allocations $\{b_{i,t+1, c_{i,t}, k_{i,t}}\}$ such that both agents maximize, the Repo

---

4The set of possible transfers allows us to choose the level of consumption at which we have full-risk sharing. For example, when $R_{U}^{US} \leq 1$, $\tau = c$ can be anywhere between 0 and $\frac{\epsilon + \pi}{2}$. For $R_{U}^{US} > 1$ that level also depends on the initial level of U.S. asset holding in the economy.

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\[ \sum_i \left( T_{i,t} - \tau \frac{1}{R_{i}^{US}} k_{i,t+1} \right) = 0. \]

We can show that a subsidy to the price of the U.S. Treasuries would allow to achieve full risk-sharing among the non-U.S. economies. This subsidy should be such that the price of U.S. assets faced by the borrowers is \( \beta \).

**Proposition 5** Let \( \tau^* = 1 - \beta R_{US}^U \), then the economy achieves full risk-sharing.

**Proof.** Similar to the proof of Proposition 4 but replacing (21) and (22) by

\[ -\frac{1}{R} u'(c) + \beta u'(\tau) + \lambda = 0 \]

\[ -\frac{(1 - \tau)}{R_{US}} u'(c) + \beta u'(\tau) + m\lambda = 0 \]

\[ -\frac{1}{R} u'(c) + \beta u'(\tau) + \lambda = 0 \]

7 Conclusion

In this paper we proposed a potential new explanation to foreign demand for U.S. Treasuries: a demand for collateral. We solved a model to highlight the mechanism and disentangle the demand for collateral from the demand for safe assets. Countries that buy U.S. Treasuries to store value differ from countries buying U.S. Treasuries for their collateral properties by how active are their repo borrowings. Finally, we used the model to study taxes on international borrowing and capital flows to achieve risk sharing. We showed that risk sharing demands either a subsidy on repo borrowings that is inversely proportional to the collateral margin and to the yield of U.S. Treasuries. Or a subsidy on the price of U.S. Treasuries.
References


Cao, D.: 2011, "Collateral Shortages, Asset Price and Investment Volatility with Heterogeneous Beliefs".


Appendix. Stochastic Economy

Consider the endowment economy in Section 3. Now we populate the economy with a continuum of agents. Each of them face idiosyncratic endowment shocks

\[ e_{i,t} = \begin{cases} \tau & \text{with probability } p \\ \varepsilon & \text{with probability } 1 - p \end{cases}. \]

Agent \( i \) solves

\[
\max_{c_{i,t}, k_{i,t+1}, b_{i,t+1}} \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t u(c_{i,t}) \right]
\]

subject to

\[ c_{i,t} + \frac{1}{R_t} k_{i,t+1} + \frac{1}{R_{US}} b_{i,t+1} \leq e_{i,t} + b_{i,t} + k_{i,t} \]

and

\[ k_{i,t+1} \geq 0 \]
\[ b_{i,t+1} \geq -m k_{i,t+1}. \]

In the case of over-collateralization, that is \( m > 1 \), we need to impose the additional natural borrowing limit. For example, in a stationary equilibrium in which \( R_t = R \) constant overtime, we must impose

\[ b_{i,t+1} + k_{i,t+1} \geq -\frac{\varepsilon}{R_{US} - 1}. \]

The numerical solution of this model gives similar qualitative results as the deterministic model of Section 3.