0.1 Objectives and Outline

A striking empirical feature of the expansion of financial institutions in Spain is the degree to which some banks and cajas expanded concentrically around their home provinces, suggesting that the cost to enter a new province decreases in the distance from a bank’s original branches. This document outlines an approach to estimating the fixed costs associated with bank expansion via a method of simulated maximum likelihood similar to the procedure suggested by Bajari, Benkard, and Levin (Econometrica, Sep 2007, 1331-1370). A first look at the data suggests that these expansion costs may be substantially heterogeneous across banks, causing some to expand only to neighboring provinces while allowing others to jump across Spain. Understanding these costs, and origins of their heterogeneity, is thus an important component to understanding the barriers to free entry in the financial system and hence the degree of market power that financial institutions may enjoy.

1 Model

In this section we first develop a parsimonious model of banks’ profit functions and entry decisions, which we later employ in the structural estimation of banks’ fixed costs. The model presented here treats banks’ interest rate decisions as the outcome of a static Nash game, but models the bank’s expansion decisions as a single agent problem faced with an aggregate number of competitors that evolves according to a reduced form Markov process. In particular, we do not yet impose that the banks’ aggregate province entry activity must be consistent with the sum of individual estimated strategies.

Banks operate chains of branches that make loans and collect deposits, earning bank $i$ a profit of $\pi_{ipt}$ from province $p$ in year $t$. Banks maximize their dynamic profits by entering new provinces and opening new branches in provinces in which they have established operations. However, before beginning operations in a new province banks must pay a fixed entry costs and must pay an
additional start-up cost to open a new branch. If banks close branches, they receive some liquidation value of the stock and building.

Defining notation, let the vector of state variables for a given bank/province/year observation be $s_{ipt}$, which includes factors such as the province’s GDP, number of own and rival branches, distance from the bank’s existing branch network, etc and let the vector $s_{it}$ contains the full set of states for all provinces in a given year. The bank’s decision to enter a province is captured by the indicator variable $\iota_{ipt}$, and the number of branches it opens/closes is $\eta_{ipt}$; the vectors $\iota_i$ and $\eta_i$ contain the full set of these policy decisions for bank $i$. The solution to the bank’s dynamic problem can be written as the two vectors $\iota_i$ and $\eta_i$ that solve

$$V(s_{it}) = \max_{\eta_i, \iota_i} \left\{ \sum_{t=\tau}^{\infty} \beta^{t-\tau} \left( \sum_{p=1}^{P} \pi(s_{ipt}) + C^P(\iota_{ipt}|s_{it}) + C^B(\eta_{ipt}|s_{it}) \right) \mid s_{i\tau} \right\}$$  

(1)

where $C^P(\iota_{ipt}|s_{it})$ is the cost the bank pays if it enters the province in year $t$, and $C^B(\eta_{ipt}|s_{ipt})$ is the cost incurred to open $\eta_{ipt}$ branches in that province.

We seek to test the hypothesis that this province entry cost $C^P_{ipt}$ is a function of the distance of that province from to the bank’s existing network of branches in other provinces.

1.0.1 Timing:

Within each period (lasting one year), the timing proceeds as follows:

1. Banks choose interest rates at the national level according to Nash equilibrium.

2. Banks make loans and take deposits in the provinces in which they are currently operating, and collect their within-period profits from these.

3. Banks’ private cost shocks to province entry and new branch opening are realized and observed by the banks.

4. Banks make decisions on which new provinces to enter, and how many branches to open or close in the provinces in which they are currently operating or plan to enter in the next year. They incur start up costs in the current period for both entering a new province and opening a new branch, although these new branches are not opened until the next period.

5. The province level GDP evolves exogenously.

6. Discounting occurs

1.1 Static Actions

Within-Period Static Profits: Banks earn profits by lending money in each of the four sectors indexed by $z$, accepting deposits at rate $r_{dep,t}$ and borrowing from the interbank market at rate $\rho$. 
Within-period profits of bank $i$ in period $t$, if it is active in province $p$ are:

$$\pi (s_{ipt}) = \sum_{z=1}^{4} l_{iptz} (r_{zt} - \rho_t) + d_{ipt} (\rho_t - r_{dep,t}) - AC_t \cdot n_{it}$$

where $l_{iptz}$ is the quantity of loans that the bank makes in sector $z$ in that province/year, $d_{ipt}$ is the total deposits in that province, and the $AC_t$ is the (assumed known to the econometrician) fixed cost of operating each of the bank’s branches. For simplicity we assume no default and that all loans are repaid fully within a year.

**Demand for Loans and Supply of Deposits** Demand for bank $i$’s loans in sector $z$ are a function of number of its branches $n_{ipt}$, the number of its competitors’ branches $n_{-ipt} = \sum_{b=1}^{B} n_{bpt}$, the interest rate for that sector, the GDP of the province, and a province/bank/sector level fixed effect $\delta_{ipz}$:

$$l_{iptz} = \beta_{1z} n_{ipt} + \beta_{2z} n_{ipt}^2 + \beta_{3z} n_{-ipt} + \beta_{4z} n_{ipt}^2 + \beta_{5z} r_{zt} + \beta_{6z} GDP_{pt} + \beta_{7z} \sum_{b=1}^{B} n_{bpt} r_{zbt} + \delta_{ipz} + \epsilon_{iptz} \tag{2}$$

The demand function for deposits is identical.

**Static Interest Rate Equilibrium** At the beginning of each period, banks play a static Nash pricing game to set interest rates at the national level. A bank that is active in a set $\mathcal{P}_i$ of provinces chooses its national interest rate for loans of type $z$ through the optimization problem,

$$\max_{r_{zt}} \sum_{\mathcal{P}_i} l_{iptz} (r_{zt} - \rho_t) + d_{ipt} (\rho_t - r_{dep,t}) - AC_t \cdot n_{it}$$

simultaneously with the $B$ other banks. The form of the demand function in equation 2 ensures that the bank’s interest rate decisions across industries are independent.

### 1.2 Dynamic Actions

**Entry Into a Province** Every year, banks decide whether to begin operations in each province in which they currently have no presence. If the bank chooses to enter a province in the next period (in which case $\iota_{ipt} = 1$), it incurs a cost $C_{itp}$, expressed as a function of the distance to other provinces in which the bank is already operating:

$$C^P (\iota_{ipt} | s_{ipt}, v^P; \gamma) = \nu_{ipt} \cdot \left( \gamma_0 + \gamma_1 dist_{ipt} + \gamma_2 v^P_{ipt} \right)$$

where $\nu^P_{ipt}$ is a $N(0,1)$ cost shock to entry. The measure of distance from the bank’s existing network, $dist_{ipt}$ is modeled as:

$$dist_{ipt} = \sum_{m \in \mathcal{P}_i, m \neq p} \frac{n_{imt}}{kms(p,m)} \tag{3}$$
where $kms(p,m)$ is the distance in kilometers between provinces $p$ and $m$.

**Opening Branches** Each period that a bank plans to operate in a province it must decide how many branches to open or close. Opening a new bank branch has a private cost, while closing branches yields a liquidation value. These are parametrized as a function of the number of new branches to be opened/closed ($\eta_{ipt}$), a random shock, and the coefficient vectors $\alpha^o$ (for opening branches) and $\alpha^c$ (for closing branches).

$$C^B(\eta|s_{ipt},\nu^B_{ipt};\alpha) = I(\eta > 0) \left( \alpha^o_0 \eta + \alpha^o_1 \nu^B_{ipt} \right) + I(\eta < 0) \left( \alpha^c_0 \eta + \alpha^c_1 \nu^B_{ipt} \right)$$

Here again $\nu^B_{ipt}$, the branch shock, is distributed $N(0,1)$. This shock enters into both the cost of opening new branches and the liquidation payoffs of closing branches. If $\alpha^o_1$ is negative and $\alpha^c_1$ is positive, a large positive realization of $\nu^B_{ipt}$ makes opening an additional branch more costly for the bank while simultaneously increasing the liquidation payoff from closing branches. Negative shocks have the opposite effect, decreasing the cost of branch openings and making closures more expensive for the bank.

2 Estimation

We denote the full vector of model parameters by $\theta = (\beta_1, ..., \beta_7, \gamma_0, \gamma_1, \gamma_2, \alpha^o_0, \alpha^o_1, \alpha^c_0, \alpha^c_1)$. This includes $\beta$, the vector of coefficients of the loan demand/deposit supply functions, $\gamma$, the vector of coefficients determining the entry cost into a new province, and $\alpha$, the vector of coefficients determining the cost to open and close branches.

2.1 Static and Reduced Form Estimation

**State Transitions:** We estimate a reduced form Markov process for the evolution of the two state variables, the number of other bank’s branches and the province GDP. In the case of GDP, $GDP_t$ depends only on $GDP_{t-1}$. For the number of competitor’s branches, we predict $n_{-ipt}$ from an OLS regression of $n_{-ipt}$ on polynomial terms of $n_{-ipt}(t-1)$, log $GDP_{(t-1)}$, and $n_{ipt}(t-1)$.

**Demand function:** The $\beta$ parameters of the demand functions for loans and deposits can be estimated using static methods. We use instrumental variables with fixed effects, using lagged values of the number of own and competitors’ branches to instrument for current period levels.

**Policy Functions** Banks’ decisions to enter provinces and construct branches are likely to be complex functions of the state variables, and solving for these policy functions at each iteration of the estimation would render the procedure unfeasibly time consuming. Instead, we can estimate
them based on the banks’ observed actions. For instance, the bank’s decision of whether to begin
operations in a new province can be estimated as a non-parametric function of the relevant state
variables,
\[ \Pr \left( \nu_{ipt}^P = 1 | s_{it} \right) = F \left( n_{-ipt}, GDP_{ipt}, kms_{ipt} \right) \]
where, as before, \( \nu_{ipt} = 1 \) is an indicator equal to 1 if the bank begins operations in province
\( p \) for the first time in period \( t \), and \( F (\cdot) \) is a flexible functional form. For now we parametrize
\( F (\cdot) \) as a logit regression on third-order polynomials of the state variables. A potential concern
in the estimation of these policy functions lies in the number of state variables to include in these
regressions. Because banks consider their full forward expansion paths when deciding to enter a
province, the characteristics of all surrounding provinces may also be included among the state
variables, thus potentially increasing them to an unfeasibly large dimension.

Likewise, we can estimate the bank’s choice of how many new branches to open or close in
a province as a flexible function of its current number of branches, the number of rival banks’
branches, and the GDP of the province:
\[ \mathbb{E} [ \eta_{ipt} | s_{ipt} ] = H \left( n_{ipt}, n_{-ipt}, GDP_{ipt} \right) \]

We estimate \( H (\cdot) \) via an ordered probit using third order polynomials of the state variables as
dependent variables.

### 2.2 Dynamic Parameters

The final two sets of parameters, \( \alpha \) and \( \gamma \), relate to the fixed costs incurred by the bank to build
branches and enter provinces. These are not identified by the banks’ static decisions since they
involve trade-offs between short term costs and long term profits from increased loans in new
provinces. In estimating these we follow a modified version of the approach of BBL’s estimation
technique: Let \( V_i \left( s_{it} | \sigma_{it}; \theta \right) \) denote the expected current and future profits of the firm if it follows
its actual strategy , \( \sigma_{it} \) (see equation 1) and \( V_i \left( s_{it} | \sigma'_{it}; \theta \right) \) be the expected value of following the
counterfactual strategy, \( \sigma'_{it} \). Given the entry shock that the firm received and at the true parameter
vector \( \theta^0 \), it must be the case that
\[ V_i \left( s_{it} | \sigma_{it}; \theta^0 \right) \geq V_i \left( s_{it} | \sigma'_{it}; \theta^0 \right) \]

Our empirical strategy is to generate estimates of the \( V_i \left( s_{it} | \sigma_{it}; \theta \right) \) and \( V_i \left( s_{it} | \sigma'_{it}; \theta \right) \) terms using
forward simulation, and then find the parameter vector \( \theta \) that maximizes the probability that the
inequality above holds for all province entry decisions that the bank makes.

**Forward Simulation** The estimation of the static demand for loans, state transition functions
and the bank’s policy functions have given us enough information to simulate paths of actions taken
by the bank:
1. Starting from \( s_{i(t+1)} \), the state of the bank after it has made the entry decision, we draw shocks \( \nu^P_{p(t+1)} \) and \( \nu^B_{p(t+1)} \) from the \( N(0,1) \) distribution.

2. For each province in which the bank is not operating we predict whether or not the bank will begin operations there by testing whether \( \Phi\left(\nu^P_{ip(t+1)}\right) > \hat{P}\left(n_{-ip(t+1)}, GDP_{ip(t+1)}, dist_{ip(t+1)}\right) \). If so, we set \( \hat{\nu}_{ip(t+1)} = 1 \).

3. For each province in which the bank will operate next year, we predict how many branches the bank will open/close by evaluating

\[
\hat{n}_{ip(t+1)} = \hat{H}\left(n_{ip(t+1)}, n_{-ip(t+1)}, GDP_{ip(t+1)}, \nu^B_{p(t+1)}\right)
\]

4. We update the GDP, interest rates, and number of other banks’ branches according to their transition functions. This generates \( s_{i(t+2)} \), and using this new state we return to step one and continue the forward simulation process.

This forward simulation continues until the discounting has reduced the value of any profit the bank makes to an insignificant amount. We use 60 periods.

### 2.2.1 Dynamic Parameter Estimation

Assume, for concreteness, that the bank’s true strategy in period \( t \) \( (\sigma_{it}) \) is not to enter province \( p \). Then the bank will follow this strategy unless the counterfactual entry strategy \( \sigma'_{it} \) if the expected future returns from this strategy are larger than the profits from the entry strategy plus the entry costs and costs of opening branches in the new province

\[
C^P\left(\nu_{ipt}; \gamma\right) + C^B\left(\nu_{ipt}; \eta_{ipt}; \alpha\right) + \beta E \left[ V_i (s_{i(t+1)} | \sigma'_{it(t+1)}; \theta) \right] \leq \beta E \left[ V_i (s_{i(t+1)} | \sigma_{it(t+1)}; \theta) \right]
\]

where the current period profit \( \pi (s_{ipt}; \beta) \) and costs of entry or branch construction in other provinces have canceled from both sides of the inequality. This relies on the assumption that province entry costs depend only upon the existing set of provinces in which the bank operates, and not on the other provinces into which the bank is also entering in a given period. Relaxing this assumption would substantially complicate the analysis since it would require analyzing all permutations of entry combinations. These would be necessary to capture the case in which, for example, isolated entry into provinces A and B is not profitable, but joint entry into A and B might be.

Rearranging yields the probability of entry,

\[
Pr (\nu_{ipt} = 1 | s_{it}; \theta) = Pr \left( \gamma_2 \nu^P_{ipt} \geq \left( -\gamma_0 - \gamma_1 dist_{ipt} - \alpha_0 \eta_{ipt} - \alpha_1 \eta_{ipt} \nu^B_{ipt} + \beta E \left[ V_i (s_{i(t+1)} | \sigma'_{it(t+1)}; \theta) \right] - V_i (s_{i(t+1)} | \sigma_{it(t+1)}; \theta) \right) \right)
\]

Empirically evaluating this expression is complicated by the fact that it contains two sources of uncertainty: the first in the future profits from entry, and the second in the current value of
the branch construction shock and hence the quantity and cost of branches constructed (which is known to the bank, but unobserved to the econometrician). The expected difference in future profits can be expressed as the average difference between two sets of forward simulations beginning from period $t$: one in which the province is entered in the first period, $\frac{1}{M} \sum_{m=1}^{M} W_{i,m} (s_{it}|\sigma_{it}; \theta)$, and one in which it is not, $\frac{1}{M} \sum_{m=1}^{M} W_{i,m} (s_{it}|\sigma'_{it}; \theta)$. By taking the mean of this difference over $M$ forward simulation paths we can estimate the difference in the bank’s expected payoffs between the two strategies. These differences must then be integrated over the second source of uncertainty: the current period shocks to the cost of opening branches in the new province. Since these are known to the bank, the bank also chooses the number of new branches to be opened accordingly, they create a joint distribution of shocks and branch openings $g(\eta_{ipt}, \nu^B_{ipt})$

$$Pr (t_{ipt} = 1|s_{it}; \theta) = 1 - \int \Phi \left( \frac{1}{M} \sum_{m=1}^{M} \left( -\gamma_0 - \gamma_1 dist_{ipt} - \alpha^0_0 \eta_{ipt} - \alpha^0_1 \eta_{ipt} \nu^B_{ipt} \right. \right. \left. \left. + W_{i,m} (s_{it}|\sigma_{it}; \theta) - W_{i,m} (s_{it}|\sigma'_{it}; \theta) \right) / \gamma_2 \right) dg (\eta_{ipt}, \nu^B_{ipt})$$

In practice, we use the first period shock draws and resulting predicted branch opening decisions to integrate over combinations of branches/shocks:

$$Pr (t_{ipt} = 1|s_{it}; \theta) = 1 - \frac{1}{M} \sum_{m=1}^{M} \Phi \left( \left( -\gamma_0 - \gamma_1 dist_{ipt} - \alpha^0_0 \eta_{ipt,m} - \alpha^0_1 \eta_{ipt,m} \nu^B_{ipt,m} \right. \right. \left. \left. + \frac{1}{M} \sum_{m=1}^{M} (W_{i,m} (s_{it}|\sigma_{it}; \theta) - W_{i,m} (s_{it}|\sigma'_{it}; \theta)) / \gamma_2 \right) \right)$$

**Maximum Likelihood**  We then set up a likelihood function:

$$\max_{\theta} \prod_t \prod_p \left( Pr (t_{ipt} = 1|s_{it}; \theta)^{t_{ipt}} (1 - Pr (t_{ipt} = 1|s_{it}; \theta))^{(1-t_{ipt})} \right) \quad (4)$$

and maximize to get $\theta$. An important simplification inherent in this likelihood function is the restriction of counterfactual strategies to only single-province deviations, for instance entering a single province that was not, in reality, entered, or not entering a single province that was entered. This simplification rules out forward looking strategies in which the bank might prefer to enter several provinces simultaneously but not individually in order to substantially reduce future entry costs into yet another set of provinces. These strategies are possible under the distance cost specification introduced in equation 3, although an alternative specification of a binary cost of distance depending on whether a province as adjacent to another in which the bank is currently operating would rule them out.