How do Households Respond to Income Shocks?*

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Abstract

We use the Italian Survey of Household Income and Wealth from 1987 to 2008 and the two most recent waves of the Panel Study of Income Dynamics (for the years 2004 and 2006), the only available micro data-sets that contain a household panel on detailed categories of income, consumption and wealth, to document co-movements of several components of the household budget constraint with short run and long run changes in labor income. We construct a measure of labor income shocks and show that these shocks are associated with modest consumption changes (that are increasing over the horizon of the change) and larger wealth changes (that are falling over the horizon of the change). The consumption evidence suggests that labor income shocks have a modest persistence and can be insured well using simple uncontingent borrowing and saving. The wealth evidence suggests an important role for wealth shocks, possibly correlated with income shocks, in affecting consumption.

Ijel codes: D91, E21

Key words: Consumption, Wealth, Risk Sharing, Precautionary Saving, Incomplete Markets

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1 Introduction

In micro-founded macro models households face one fundamental decision problem, namely how to choose consumption and saving in the presence of both deterministic labor income changes as well as labor income shocks. The feasible consumption-savings choices of households crucially depend on the menu of financial and real assets available to them. Existing models differ starkly with respect to the assumptions regarding this menu. At one extreme, in so-called hand-to-mouth consumer models financial assets are entirely absent and consumption bears all the adjustment to income shocks. In the other extreme, the complete markets model (the underlying abstraction of any representative agent macro model) envisions a full set of state contingent assets that households can trade without binding short sale constraints. In this model wealth bears all the adjustment to an income shock, and consumption bears none (unless the income shock is an aggregate shock and cannot be diversified internationally).

The assumptions the model builder makes about the structure of financial markets are crucial not only for the positive predictions of the model (e.g. the joint income-consumption dynamics, the response of the macro economy to shocks, the pricing of financial assets) but also for normative policy analysis. The desirability of social insurance policies (e.g. unemployment insurance, a redistributive tax code) depend crucially on how well households can privately (self-) insure against idiosyncratic income shocks, which in turn is determined by their access to and the sophistication of asset markets. Thus, it is important to determine empirically what actual households do when they receive income shocks, and to use this evidence to determine which consumption-savings model provides the best approximation to this observed behavior. The importance of the question has indeed generated a lot of work on this issue but which model fits best actual household response remains unclear. Most authors have focused on the consumption response to income shocks alone, and have not explicitly analyzed the corresponding wealth response, mostly due to the lack of suitable panel data on income, consumption and wealth. This paper is, to the best of our knowledge, the first attempt to empirically evaluate the predictions, with respect to the consumption and wealth response of households to income shocks, of alternative consumption-savings models that uses data for the same households on changes in income, consumption and wealth, both in the short and in the long run.

To carry out our analysis, we use the only two panel datasets that contain detailed information about household income, consumption and wealth: the Italian Survey of Household Income and Wealth (SHIW) for the years 1987-2008 and the last two waves (2004 and 2006)
of the U.S. Panel Study of Income Dynamics (PSID).\(^1\)

Our analysis documents co-movements, at the household level, of labor income with other components of income, with various component of consumption and wealth for the whole sample of households who have at least one member who is between the age of 25 and 55, and is not retired. The analysis suggests that is useful to divide households in two groups: households that own businesses or real estate and households that do not. In the SHIW we find that for households who do not own business wealth nor real estate nondurable consumption changes by about 23 cents in response to a short run (two years) 1 Euro change in after-tax labor income, while financial wealth responds by about 17 cents. Remarkably, we find a quantitatively very similar response in the PSID. In the SHIW we also find that in response to longer run (six years) income changes the consumption response becomes stronger, while the wealth response becomes weaker. For households who own real estate or businesses we find that the consumption response to income shocks is much smaller (in the order of 5 cents to the dollar) while the wealth response is considerably larger.

We then explore whether various versions of a standard incomplete markets model can account for these empirical findings. We first evaluate the simplest variant of such a model, a formalized version of the permanent income hypothesis, in which households can freely borrow and save with a risk-free bond whose real return equals the subjective household time discount rate, face no binding borrowing constraints, have quadratic utility and face both purely transitory and purely permanent shocks. In that model one can derive the consumption and wealth responses to an income shock analytically and show that they are simple functions of the ratio between the variance of the transitory and the permanent shock, as well as the share of the transitory shock that is due to measurement error in income. We show that the co-movement between income, consumption and wealth changes both in the short run and in the long run predicted by the model is quantitatively consistent with that observed in the data for non-business, non-real estate owners, if transitory shocks are an important source of income changes and if measurement error in income is substantial. As we argue in the paper, we believe that the relative magnitude of transitory income shocks and measurement error required for the model to fit the data well is of plausible magnitude, and therefore conclude that the simple PIH model does very well in explaining the observed consumption and wealth responses of non-business, non-real estate owners in the short run.

We next show that, in the context of the standard incomplete markets model, the long run wealth response to an income shock is particularly informative about the nature of

\(^1\)The last two waves are the only ones which contain comprehensive information on income, consumption and wealth of each household
the precautionary savings motive. In models in which the size of the precautionary saving motive is independent of the income realization or the wealth level of the household (such as the pure PIH model or a model with CARA utility and nonbinding borrowing constraints\textsuperscript{2}) the wealth response to an income shock should be falling with the time horizon of an income change (i.e. the wealth response to a 1 Euro income change over two years should be stronger than the response to a 1 Euro income change over six years). In contrast, in versions of the incomplete markets model in which households have CRRA utility and/or face borrowing constraints we will show, using numerical simulations, that the wealth response to an income shock should be increasing with the time horizon. Therefore the empirical evidence that the wealth responses to income shocks weakens with the time horizons suggests that the income and wealth dependent precautionary savings motive implied by the CRRA model does not receive empirical support from our Italian data. Instead, also along this dimension the empirical findings are consistent with the pure PIH.

We then analyze the wealth response to income shocks in more detail and document that, of all components of wealth, the value of real assets (especially real estate and businesses) co-moves especially strongly with labor income shocks, for the whole sample of households. We argue that a large part of this co-movement may be driven by a strong correlation between labor income shocks and the prices of real estate (respectively, the value of businesses), rather than reflect wealth accumulation behavior of households in response to labor income shocks. This leads us to conclude that a simple model in which households only face idiosyncratic income shocks, but not shocks to the value of their assets, might only be a good approximation for households that do not own real estate or businesses, but not for the entire sample of households. This conclusion in turn motivates our sample selection in the first part of the paper.

The paper is organized as follows. In the next section we briefly place our contribution into the existing empirical and theoretical-quantitative literature. The data we use as well as the empirical results we derive are discussed in section 3. In section 4 we present and evaluate simple partial equilibrium versions of incomplete markets consumption-savings models against the empirical facts documented in section 3. Section 5 presents further evidence on the importance of adjustments in the value of real estate and business wealth associated with labor income shocks, and section 6 concludes.

\textsuperscript{2}In the PIH model there is no precautionary saving at all. In a model with CARA utility, absent borrowing constraints, households engage in precautionary saving, but the amount they save for precautionary reasons is independent of their income or wealth level, and the realization of their income shock. Thus the PIH and the CARA utility version of the incomplete markets model have exactly the same predictions for the consumption response to an income shock (and thus exactly the same predictions for the regression coefficients we estimate). For a full theoretical treatment of the CARA case see e.g. Wang (2003).
2 Related Literature

This paper builds on the large literature that has used household level data sets to quantitatively evaluate or formally statistically test the empirical predictions of Friedman’s (1957) permanent income hypothesis and related partial equilibrium incomplete markets models. Hall and Mishkin (1982) and Altonji and Siow (1987) represent seminal contributions, and the early body of work is discussed comprehensively in Deaton (1992). How strongly consumption responds to income shocks of a given persistence is the central question of this literature.\(^3\)

How strongly consumption responds to income shocks has also been estimated, for the U.S., in the context of tests of perfect consumption insurance, see e.g. Mace (1991), or Cochrane (1991). These tests do not need to distinguish between expected income changes and income shocks, and between transitory and permanent shocks since all income fluctuations ought to be smoothed and all shocks fully insured, according to the null hypothesis of perfect consumption insurance.

Dynarski and Gruber (1997) and Krueger and Perri (2005, 2006) take a more agnostic view about the underlying model generating the data and present the correlation between income and consumption changes as a set of stylized facts that quantitative models ought to match. The spirit of our empirical analysis is similar to these studies. For Italy, in a sequence of papers Jappelli and Pistaferri (2000, 2006, 2010a, 2011) employ the SHIW data to study the dynamics of household income, and the latter three the joint dynamics of household income and consumption.\(^4\)

Recently Blundell et al. (2008) have constructed a consumption and income panel by skillfully merging data from the CEX and the PSID, and used this panel to estimate the extent to which households can insure consumption against transitory and permanent income shocks. Kaplan and Violante (2011) evaluate whether a class of incomplete markets models can rationalize the empirical estimates for consumption insurance that Blundell et al. (2008) obtain.

Finally, Aaronson et al. (2011) investigate the consumption response to an increase in the real wage in the U.S. Similar to our study they find that the adjustment in real estate wealth is a crucial feature in their data, and they construct a model with housing wealth to account for the facts.

\(^3\)How strongly households’ consumption responds to predictable changes in income is the subject of studies on excess sensitivity. The excess smoothness literature studies how strongly household consumption adjusts in response to permanent income shocks. See e.g. Luengo-Prado and Sorensen (2008).

\(^4\)See Padula (2004) for another empirical study that uses the same Italian data.
3 Evidence

We use the only two panel datasets (to the best of our knowledge) that contain detailed information about household income, consumption and wealth: the Italian Survey of Household Income and Wealth (SHIW) for the years 1987-2008 and the last two waves (2005-2007) of the Panel Study of Income Dynamics (PSID).

3.1 Survey of Household Income and Wealth

The Survey of Household Income and Wealth (henceforth SHIW) is conducted by the Bank of Italy. The survey started in 1965, but before 1987 it did not contain any panel dimension and did not contain complete wealth and consumption data. From 1987 on the SHIW has been conducted every two years (with the exception of the 1995 and 1998 surveys which were conducted 3 years apart) and it includes about 8000 households per year, chosen to be representative of the whole Italian population. Also it has a panel structure and a fraction of households in the sample is present in the survey for repeated years. This data set is valuable and unique for our purposes as it contains long panel information for many categories of income, consumption and wealth for each household. The panel dimension on income is particularly helpful for assessing the nature (i.e. permanent or temporary) of income changes and shocks. The fact that the data contains, for the same household, panel information on income, consumption and wealth is crucial for inferring how a given household adjusts its consumption in response to an income change of a given type, and which and how various components of wealth change in association with income fluctuations. Table A1 in the appendix displays the total sample size of the data as well as the share of the households in each wave of the SHIW that was present already in previous waves. We observe that the panel dimension of the data set since 1989 is substantial and has grown over time, with the fraction of all households in the 2008 wave already being present in previous waves exceeding 50%.

Since the focus of this project is on the effects of earnings changes for households who are active in the labor market we define an observation as a household who is in the survey for at least two consecutive periods and whose head is between the age 25 and 55 and is not

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5Jappelli and Pistaferri (2010a) show that aggregate consumption and aggregate income from the SHIW display growth rates that are very similar to the corresponding NIPA figures, suggesting that the coverage of the survey is comprehensive.

6The US consumer expenditure (CEX) survey has a panel dimension but the fact that it is short (only two periods), that observation periods for income and consumption do not perfectly coincide (see Gervais and Klein, 2010 for a treatment of this problem) and the fact that there is no panel dimension for wealth makes it of limited use for our purposes.
retired in both periods. This leaves us with a sample of 14272 observations over the period 1987-2008.

3.1.1 Organization of the Data and Measurement

In order to organize our empirical findings we place them into the context of a sequential budget constraint of a standard incomplete markets model in which the household can self-insure by buying and selling a limited set of assets. This budget constraint reads as:

\[ c_{nt} + c_{dt} + a_{t+1} + e_{t+1} = y_t + p_t + a_t + e_t + T_t, \]  

where \( c_{nt} \) and \( c_{dt} \) denote consumption expenditures on nondurables (including rent and imputed rent for owner occupied housing) and durable consumption, respectively. \( a_{t+1} \) and \( e_{t+1} \) denote the values of the net asset position of financial and real wealth at the end of period \( t \), whereas \( y_t \) measures after-tax labor income, \( T_t \) net private and public transfers, and \( p_t \) denote asset income, including income from financial assets (i.e. interests and dividends) and income from real wealth (rental income), correspondingly. Financial wealth includes liquid assets such as stocks and bonds while real wealth includes three types of less liquid assets i.e. real estate, ownership shares of non incorporated business and valuables (i.e. precious metals, art etc). Our Italian data is rich enough that we can measure all these variables for our households in the sample.\(^7\) The first step of our empirical analysis is to control for differences in family size across households by dividing each observation by the appropriate OECD equivalence scale.\(^8\)

Table 1 below reports some basic summary statistics for our sample.

We then denote by \( \Delta^N x = \frac{x_t - x_{t-N}}{N} \) the annualized difference between an equivalized variable \( x \) today and \( N \) periods ago and we obtain, setting \( N = 2 \) (with the exception of 1998 where we set \( N = 3 \)):

\[
\Delta^2 c_{nt} + \Delta^2 c_{dt} + \Delta^2 a_{t+1} + \Delta^2 e_{t+1} \\
= \Delta^2 y_t + \Delta^2 p_t + \Delta^2 T_t \\
+ \Delta^2 a_t + \Delta^2 e_t
\]  

\(\Delta\) is the differences operator.

\(^7\)For the exact variable definitions in the SHIW, please see Appendix A

\(^8\)This procedure has a minor impacts on the results. For labor income \( y_t \), for example, around 99% of the cross-sectional variation of equivalized income growth is due to variation in the growth rate of raw income.
Observed: $c_{it}, c_{it+1}, y_{it}, T_{it}, p_{it}$

$at_{t+1}, et_{t+1}$

Not Observed: $at, et$

Figure 1: Timeline in the SHIW

Table 1. SHIW Sample summary statistics

<table>
<thead>
<tr>
<th></th>
<th>Average Level</th>
<th>Annualized Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age of head</td>
<td>41.5</td>
<td>44.6</td>
</tr>
<tr>
<td>Household size</td>
<td>3.8</td>
<td>3.3</td>
</tr>
<tr>
<td>Labor income</td>
<td>8156</td>
<td>10646</td>
</tr>
<tr>
<td>Asset income</td>
<td>1211</td>
<td>2690</td>
</tr>
<tr>
<td>Transfers</td>
<td>285</td>
<td>563</td>
</tr>
<tr>
<td>Non Durable consumption</td>
<td>5766</td>
<td>6858</td>
</tr>
<tr>
<td>Durable consumption</td>
<td>860</td>
<td>888</td>
</tr>
<tr>
<td>Real Wealth</td>
<td>25603</td>
<td>71366</td>
</tr>
<tr>
<td>Financial Wealth</td>
<td>5124</td>
<td>9638</td>
</tr>
</tbody>
</table>

Note: All variables except age and size, are per adult equivalent and in 2000 Euros

Note that due to the biannual nature of our data set the last two terms $\Delta^2 a_t$ and $\Delta^2 e_t$ cannot be observed in the data. This fact is clarified in figure 1 which shows the frequency and exact timing with which different variables are observed in the SHIW data set.

The empirical question we want to answer now is how the observable differences in the budget constraint co-move with changes in labor income $\Delta^2 y_t$. Since our main focus is on income changes that are idiosyncratic and unpredicted (that is, on idiosyncratic income
we first attempt to purge the data from aggregate effects and predictable individual changes by regressing each change on time dummies, on a quartic in the age of the head of the household, on education and regional dummies, and on age-education interaction dummies. Our empirical exercise is then carried out on the residuals from these first-stage regressions.

3.1.2 Empirical Results

In figure 2 we display the cumulative distribution function of observed residual annualized labor income changes and log changes. The picture shows that a substantial fraction (about 20%) of households experience income changes that are larger than 2000 Euros (annualized, per adult equivalent) or larger than 20% of their labor income.

In order to assess which households are subject to larger income shocks, in figure 3 we order households with respect to residual income changes, sort them into twenty equally sized bins and for each bin we plot the fraction of households whose head is self-employed. The figure shows clearly how self-employed households experience, on average, larger absolute and relative changes.\footnote{Guiso et al. (2005) document that Italian firms provide substantial earnings insurance to its employees against firm-specific shocks. The stark difference between the earnings shocks for employees and self-employed in figure 3 could therefore partly be due to the fact that employees are partially insured by their firms against idiosyncratic (to the firm or the worker) productivity shocks.}

We fully acknowledge that a possibly large share of this observed variation in labor earnings may be due to measurement error or to components that are predictable to the household but not to us, and thus will address these issues explicitly when comparing the stylized facts from the data to the predictions of the models we use to assess these facts.\footnote{Altonji and Siow (1987), in their critique of Hall and Mishkin (1982) stress the potential quantitative importance of measurement error in income changes or income growth for the type of regressions conducted in this paper.}

To visualize the co-movement of various components of the budget constraint with income for each of the 20 bins of sorted income changes we compute the average change in each observable component of the budget constraint and plot it against the corresponding income change. Figures 4-6 contain the results of this exercise, for nondurable and durable consumption, non-labor income components and all forms of household wealth.

From figure 4 we observe that nondurable consumption changes are positively correlated with income shocks. In addition, that relationship appears to be fairly linear, although a slightly larger response to income increases than to income declines can be observed. As we make precise below in table 2, for the entire sample of households, on average a 1 Euro increase (decline) in after-tax labor income is associated with about a 10 cents increase.
Note: Income is real after tax labor + business per adult equivalent. Growth rates and changes are annualized.

Figure 2: CDF of residual income variation
Figure 3: Income variation and self employment
Figure 4: Changes in income and non durable consumption

Income and consumption changes

Changes are annualized and in thousands of 2000 Euros. Each dot averages approx. 700 households.
Figure 5: Changes in income and selected components of budget constraint

(decline) in expenditures on nondurable consumption.

In figure 5 we display the co-movement of after-tax labor income with other parts of household income, in particular transfer income (the upper right panel), and capital income from both real assets and financial assets (the lower two panels). The upper left panel shows the change in consumption expenditures on consumer durables (mainly cars and furniture) for each income change bin. We observe that changes in expenditures on consumer durables co-move positively with income shocks but less so than changes in expenditures on nondurables. Labor and capital income changes are, broadly speaking, uncorrelated with each other. On the other hand, there is a visible, significant, but quantitatively small negative co-movement between labor income changes and the change in net public and private transfers received by households. This negative correlation is especially noticeable for households with large income increases.

Figure 6 shows instead the co-movement of changes in various wealth components with labor income and shows how total wealth and all its components (financial wealth, real estate wealth and business wealth) strongly co-move with labor income.

In order to formally evaluate the magnitude of the average response of the various
Households are sorted by residual labor income changes. Each dot averages approx 700 households. Changes are annualized and in thousands of 2000 Euros.

Figure 6: Changes in income and wealth components
components of the budget constraint to income changes we now run bivariate regressions of the changes in the various component of the budget constraint on the changes in income: results are reported in tables 2 and 3 below. Since the OLS estimates, in particular for the wealth observations, may be influenced by a few large outliers that report large positive or negative changes in wealth, we also report the median regression (MR) estimates resulting from minimizing the sum of the absolute values of the residuals, rather than the sum of squared residuals. By putting less weights on extreme observations MR estimates are more robust to the influence of outliers.

Table 2. Co-movement with changes in labor income of:

<table>
<thead>
<tr>
<th></th>
<th>$\Delta c_n$</th>
<th>$\Delta c_d$</th>
<th>$\Delta T$</th>
<th>$\Delta T_P$</th>
<th>$\Delta T_O$</th>
<th>$\Delta p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{OLS}$</td>
<td>6.0</td>
<td>2.8</td>
<td>-0.9</td>
<td>-3.1</td>
<td>2.4</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>(1.33)</td>
<td>(1.98)</td>
<td>(0.40)</td>
<td>(0.75)</td>
<td>(0.85)</td>
<td>(0.37)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.03</td>
<td>0.01</td>
<td>0.00</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\beta_{MR}$</td>
<td>10.5</td>
<td>0.4</td>
<td>0.00</td>
<td>-0.07</td>
<td>0.00</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>(0.20)</td>
<td>(0.04)</td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.04)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Obs.</td>
<td>14272</td>
<td>14272</td>
<td>14272</td>
<td>7852</td>
<td>7852</td>
<td>14272</td>
</tr>
</tbody>
</table>

Note: SE clustered at household level (for OLS) are in parenthesis

Results in table 2 quantitatively confirm the visual evidence from figures 4 and 5 that changes in expenditures on consumer non durables $\Delta c_n$ and in durables $\Delta c_d$ are significantly associated with changes in income but are much smaller than the income changes. On average when income change by 1 Euro total consumption expenditures change by about 16 cents. The figure also shows that other sources of income are only weakly correlated with labor income changes. This table also splits total net transfers $T$ into transfers from family and friends $T_F$ and other transfers $T_O$ (which includes pensions and arrears) and indicates that the former accounts for the majority of the (not very large) negative correlation between labor income changes and changes in transfers. The adjustment of family transfers for a Euro in lower labor income is in the order of 4 cents. The existence and negative correlation with labor income changes of changes in family transfers may lend some qualitative support to models that permit household to engage in more explicit insurance arrangements than the simple self-insurance through asset trades that standard incomplete markets models envision (e.g. models with private information or limited commitment). Note, however, that these changes in transfers and their correlation with labor income changes are quantitatively very

11Note that the lower number of observation in the $T_F$ and $T_O$ regression is due to the fact that disaggregated data on transfers are not available in the early survey years.
small.

Table 3. Co-movement with changes in labor income of:

<table>
<thead>
<tr>
<th></th>
<th>$\Delta a$</th>
<th>$\Delta a_f$</th>
<th>$\Delta a_{re}$</th>
<th>$\Delta a_{bw}$</th>
<th>$\Delta a_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{OLS}$</td>
<td>283.9</td>
<td>13.4</td>
<td>85.5</td>
<td>183.7</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td>(70.4)</td>
<td>(10.2)</td>
<td>(41.7)</td>
<td>(34.9)</td>
<td>(1.6)</td>
</tr>
<tr>
<td>$\beta_{MR}$</td>
<td>132.0</td>
<td>15.7</td>
<td>31.8</td>
<td>29.3</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>(2.32)</td>
<td>(0.44)</td>
<td>(1.62)</td>
<td>(0.86)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.04</td>
<td>0.00</td>
<td>0.01</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>Obs.</td>
<td>14272</td>
<td>14272</td>
<td>14272</td>
<td>14272</td>
<td>14272</td>
</tr>
</tbody>
</table>

Note: SE clustered at household level (for OLS) are in parenthesis

Results in table 3 confirm the findings from figure 6 that changes in labor income are strongly associated with changes in wealth. The first column reports the result of regressing residual changes in total wealth on residual changes in labor income while the subsequent columns report the results using financial wealth ($a_f$), real estate wealth ($a_{re}$), business wealth ($a_{bw}$) and valuables ($a_v$). Notice that results change significantly whether we use OLS or MR regressions, suggesting that there are some households reporting very large changes in wealth (in particular business wealth) which strongly affect the OLS results. The upshot of the table though is that, regardless of the regression method, on average a 1 Euro change in labor income is associated with changes in wealth that are larger than 1 Euro. This result suggests that a simple consumption/saving model in which households are subject solely to income shocks cannot be consistent with this fact. We conjecture that the main reason for this result is the presence of shocks to the value of the wealth which are correlated with the value of labor income. An example of this would be an entrepreneur that receives a positive shock to the value of his business which at the same raise both her measured labor income and her wealth. Another example would be a city-specific shock which raises, at the same time, labor income and real estate wealth of its residents. Therefore, in order to isolate household response to a "pure" income shock we now select households which do not have any members who are self-employed/entrepreneur and who do not own real estate.

\footnote{Note that this large change in the real value of assets is not in principle inconsistent with the budget constraint. If income in period $t-1$ (which we do not observe, due to the biannual structure of the data set) were highly correlated with income change $y_t - y_{t-2}$ then the right hand side of the budget constraint could change more than 1 Euro for each Euro in $\Delta y$. In practice though, for empirically relevant income processes this correlation is not high enough to generate such a large response of wealth.}
Table 4. Co-movements for selected sample

<table>
<thead>
<tr>
<th></th>
<th>$\Delta c_n$</th>
<th>$\Delta c_d$</th>
<th>$\Delta T$</th>
<th>$\Delta a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{OLS}$</td>
<td>26</td>
<td>8.7</td>
<td>-7.5</td>
<td>35.6</td>
</tr>
<tr>
<td></td>
<td>(2.7)</td>
<td>(2.9)</td>
<td>(1.1)</td>
<td>(12)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.02</td>
<td>0.01</td>
<td>0.04</td>
<td>0.01</td>
</tr>
<tr>
<td>$\beta_{MR}$</td>
<td>22</td>
<td>1.5</td>
<td>-0.3</td>
<td>43.6</td>
</tr>
<tr>
<td></td>
<td>(1.1)</td>
<td>(0.3)</td>
<td>(0.1)</td>
<td>(8.3)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>Obs.</td>
<td>2932</td>
<td>2932</td>
<td>2932</td>
<td>2932</td>
</tr>
</tbody>
</table>

Note: SE clustered at household level (for OLS) are in parenthesis

The key result to notice from table 4 is that for this selected sample nondurable consumption co-moves significantly more strongly with income and significantly less with wealth. The consumption response is in the order of 22 to 26 cents for the Euro, and the response of wealth 35 to 43 cents. In the next section we now assess whether, as a first basic check of consumption theory, the standard formalized version of the permanent income hypothesis in the spirit of Friedman (1957) provides a reasonable approximation of the data for this selected group of households. This analysis also provides some guidance along what dimension this basic model ought to be extended to match the co-movements fact for the whole sample of households.

3.2 Panel Study of Income Dynamics

The Panel Study of Income Dynamics (PSID) is a longitudinal study of a sample of US families. The PSID data files provide a wide variety of information about both families and individuals, with substantial detail on income sources and amounts, employment status and history, family composition changes, and residential location. Recent changes in the PSID make it both more comparable to the SHIW and useful for our analysis. Specifically, the PSID is now collected bi-annually, as the SHIW. More importantly, the last two waves of the PSID (2004 and 2006) contain comprehensive data on consumption expenditures and on end of period wealth so that we can repeat the same exact exercise we conducted on the SHIW.
Figure 7: Consumption response in PSID

<table>
<thead>
<tr>
<th></th>
<th>Average Level</th>
<th>Annualized Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age of head</td>
<td>39.7</td>
<td>40.1</td>
</tr>
<tr>
<td>Household size</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Labor income</td>
<td>22480</td>
<td>22513</td>
</tr>
<tr>
<td>Asset income</td>
<td>661</td>
<td>680</td>
</tr>
<tr>
<td>Transfers</td>
<td>1599</td>
<td>1400</td>
</tr>
<tr>
<td>Non Durable consumption</td>
<td>15927</td>
<td>17187</td>
</tr>
<tr>
<td>Total wealth</td>
<td>79915</td>
<td>89125</td>
</tr>
</tbody>
</table>

All variables except age and size, per adult equiv. and in 2004 Dollars

4 Theory

4.1 The Permanent Income Hypothesis

We now want to investigate whether versions of a standard incomplete markets model are consistent with the facts displayed in the previous section. In this section we summarize the empirical predictions of a model based on the permanent income hypothesis for the question at hand, and evaluate to what extent the empirical evidence presented above is consistent with this model. In the next section we then study a calibrated version of a
Figure 8: Transfer response in PSID

Figure 9: Wealth response in PSID
standard incomplete markets life cycle model with a precautionary savings motive.

Suppose that households have a quadratic period utility function, can freely borrow and lend\(^{13}\) at a fixed interest rate \(r\), discount the future at time discount factor \(\beta\) that satisfies \(\beta (1 + r) = 1\) and faces an after-tax labor income process of the form

\[
\begin{align*}
y_t &= \bar{y} + z_t + \varepsilon_t + \gamma_t \\
z_t &= z_{t-1} + \eta_t
\end{align*}
\]

where \(\bar{y}\) is expected household income, \(\varepsilon_t \sim N(0, \sigma^2_\varepsilon)\) is a transitory income shock, \(\eta_t \sim N(0, \sigma^2_\eta)\) is a permanent income shock and \(\gamma_t \sim N(0, \sigma^2_\gamma)\) is classical measurement error in income. The shocks \((\varepsilon, \eta, \gamma)\) are assumed to be uncorrelated over time and across each other. where \((\varepsilon, \eta)\) are uncorrelated i.i.d. shocks with variances \((\sigma^2_\varepsilon, \sigma^2_\eta)\).

Aggregating across wealth components and focusing on nondurable consumption the household faces a budget constraint of the form

\[
c_t + w_{t+1} = y_t + (1 + r)w_t
\]

where \(w_t = a_t + e_t\) is total and \(c_t\) are expenditures on nondurable consumption, including (imputed) rent for housing. We show in the appendix how a model that includes housing explicitly can be reduced to the formulation studied in this section as long as there are competitive rental markets, and the stock of housing can be adjusted without any frictions or binding financing constraints. In addition, for the empirical implementation of this model we include transfers \(T_t\) as part of after-tax labor income.

4.1.1 Empirical Predictions

As is well known, the realized changes in income, consumption and wealth of this model are given by (see e.g. Deaton, 1992):

\[
\begin{align*}
\Delta c_t &= \frac{r}{1 + r} \varepsilon_t + \eta_t \\
\Delta w_t &= \frac{\varepsilon_t}{1 + r} \\
\Delta y_t &= \eta_t + \Delta \varepsilon_t + \Delta \gamma_t
\end{align*}
\]

where \(\Delta x_t = x_t - x_{t-1}\).

Equipped with these results we can now deduce the consumption and wealth responses

\(^{13}\)Of course a No-Ponzi condition is required to make the household decision problem have a solution.
to income changes, as measured by the same bivariate regressions we ran for our Italian
data. First, since we have available a full panel and the survey is carried out only two
periods, we need to work with changes of variables over $N$ periods, which are given by:

$$\Delta^N x_t = x_t - x_{t-N} = \Delta x_t + \Delta x_{t-1} + \ldots + \Delta x_{t-N+1}.$$ 

Using (3) we find that

$$\Delta^N c_t = \sum_{\tau=t-N+1}^{t} \left( \frac{r \varepsilon_{\tau}}{1+r} + \eta_{\tau} \right)$$

$$\Delta^N w_t = \sum_{\tau=t-N+1}^{t} \frac{\varepsilon_{\tau}}{1+r}$$

$$\Delta^N y_t = \sum_{\tau=t-N+1}^{t} \eta_{\tau} + \Delta^N \varepsilon_t + \Delta^N \gamma_t$$  

(4)

and thus the bivariate regression coefficients of $N$-period consumption and wealth changes
on $N$-period income change are given as

$$\beta^N_{c} = \frac{Cov(\Delta^N c_t, \Delta^N y_t)}{Var(\Delta^N y_t)} = \frac{Cov\left(\sum_{\tau=t-N+1}^{t} \left( \frac{r \varepsilon_{\tau}}{1+r} + \eta_{\tau} \right), \sum_{\tau=t-N+1}^{t} \eta_{\tau} + \Delta^N \varepsilon_t + \Delta^N \gamma_t \right)}{Var\left(\sum_{\tau=t-N+1}^{t} \eta_{\tau} + \Delta^N \varepsilon_t + \Delta^N \gamma_t \right)}$$

$$= \frac{N \sigma^2_\eta + r \sigma^2_\varepsilon}{N \sigma^2_\eta + 2 (\sigma^2_\varepsilon + \sigma^2_\gamma)}$$

$$\beta^N_{w} = \frac{Cov(\Delta^N w_t, \Delta^N y_t)}{Var(\Delta^N y_t)} = \frac{Cov\left(\sum_{\tau=t-N+1}^{t} \frac{\varepsilon_{\tau}}{1+r}, \sum_{\tau=t-N+1}^{t} \eta_{\tau} + \Delta^N \varepsilon_t + \Delta^N \gamma_t \right)}{Var\left(\sum_{\tau=t-N+1}^{t} \eta_{\tau} + \Delta^N \varepsilon_t + \Delta^N \gamma_t \right)}$$

$$= \frac{\sigma^2_\varepsilon}{(1+r) \left[ N \sigma^2_\eta + 2 (\sigma^2_\varepsilon + \sigma^2_\gamma) \right]}.$$

Conditional on a real interest rate $r$ these regression coefficients can be expressed ex-
clusively as functions of the ratio of the size of permanent to transitory shocks $Q = \frac{\sigma^2_\eta}{\sigma^2_\varepsilon + \sigma^2_\gamma}$

and the share of transitory income shocks attributed to measurement error$^{14}$, $M = \frac{\sigma^2_\gamma}{\sigma^2_\varepsilon + \sigma^2_\gamma}$.

$^{14}$The estimated coefficient $\beta^N_c$ can be decomposed into the regression coefficient obtained if income was
measured without error, $\beta$, and the attenuation bias stemming from measurement error:

$$\beta^N_c = \beta \ast \frac{1}{1 + N \sigma^2_\eta / 2 \sigma^2_\varepsilon}$$
Using these definitions we find

\[ \beta_c^N = \frac{NQ + (1 - M) \frac{r}{1+r}}{NQ + 2} \]  

\[ \beta_w^N = \frac{1 - M}{(1 + r) [NQ + 2]} \]  

Straightforwardly, the larger is the size of the permanent shock, relative to the transitory shock, as measured by \( Q \), the larger is the consumption response \( \beta_c^N \) and the smaller the wealth response \( \beta_w^N \). Second, increasing the period length \( N \) acts exactly like an increase in \( Q \) (notice that \( N \) and \( Q \) appear in the expressions above as a product exclusively). Transitory shocks are mean-reverting of the horizon of \( N \) years, whereas all permanent shocks during the \( N \) year accumulate in income income changes, see equation (4). Therefore an increase in \( N \) effectively increases the persistence of income shocks, and thus the PIH implies that the coefficient \( \beta_c^N \) is increasing in \( N \) and \( \beta_w^N \) is decreasing in \( N \). To evaluate this last prediction in particular requires panel data on labor income, consumption and wealth, which the Italian data, uniquely among household level data sets for industrialized countries, provides.

Larger measurement error lowers both coefficients due to the standard attenuation bias: it increases the variance of observed income, but leaves consumption and wealth unaffected since it is only income variation observed by the econometrician, but not experienced by the household. From equation (5) we observe that the share of measurement error is quantitatively unimportant for \( \beta_c^N \) for plausible values of \( r \). True transitory shocks to income translate into consumption with a factor \( \frac{r}{1+r} \approx 0 \), while measurement error has an impact of exactly 0. Thus, to a first approximation the share \( M \) of measurement error does not affect \( \beta_c^N \). On the other hand, true transitory income shocks translate into changes in wealth one

where

\[ \beta = \frac{\text{Cov} \left( \sum_{t=t-N+1}^{t} \left( \frac{\varepsilon_t}{1+r} + \eta_t \right) \right) \cdot \sum_{t=t-N+1}^{t} \eta_t + \Delta^N \varepsilon_t}{\text{Var} \left( \sum_{t=t-N+1}^{t} \eta_t + \Delta^N \varepsilon_t \right)} \]

\[ = \frac{N\sigma^2_\eta + r\sigma^2_\varepsilon/(1+r)}{N\sigma^2_\eta + 2\sigma^2_\varepsilon} \]

so that

\[ \beta_c^N = \frac{N\sigma^2_\eta + r\sigma^2_\varepsilon/(1+r)}{N\sigma^2_\eta + 2\sigma^2_\varepsilon} \cdot \frac{1}{1 + \frac{2\sigma^2_\varepsilon}{N\sigma^2_\eta + 2\sigma^2_\varepsilon}} \]

\[ = \frac{N\sigma^2_\eta + r\sigma^2_\varepsilon/(1+r)}{N\sigma^2_\eta + 2\sigma^2_\varepsilon + 2\sigma^2_\varepsilon} \]

We observe how the size of the bias in \( \beta_c^N \) is decreasing in \( N \) and \( Q \). Thus another useful aspect of the longer panel dimension of the Italian data set is that it allows us to use income changes over longer time periods which mitigates the problem of (classical) measurement error in income.
for one, whereas measurement error does not have any impact on the changes in wealth. Therefore the degree of measurement error $M$ has a strong impact on $\beta_{Nw}^N$, as (6) shows.

Finally, we observe that the size of the income innovations, $\sigma_z^2, \sigma_n^2$ per se has no impact on the regression coefficients. This is to be expected since quadratic utility and the absence of binding borrowing constraints implies that the household consumption and wealth choices obey certainty equivalence, and a precautionary saving motive is absent. In the next subsection we will evaluate how important the incorporation of a precautionary savings motive is to rationalize the empirically observed co-movement of labor income, consumption and wealth.

### 4.1.2 Evaluating the Empirical Predictions

We now ask whether for the sample of households that we identified in the empirical section as most appropriately modeled by the PIH, households without business and real estate wealth, the PIH is consistent with data. First, we let $N = 2$ and look at the minimal panel dimension, which in turn contains the maximal number of households in the data. For concreteness, we assume a real interest rate of $2\%$. Equations (5)-(6) show that the exact value of the real interest rate affects the predicted values for $(\beta_{2c}^2, \beta_{2w}^2)$ only insignificantly. We then ask what values of $Q, M$ are needed to assure that the model predicts the same regression coefficients as in the data.

Recall that the empirical regression results for the subsample under question delivered a consumption response of $\beta_{2c}^2 = 0.23$ and a financial wealth response of $\beta_{2w}^2 = 0.17$. Using equations (5)-(6) we can determine which degree of income persistence $Q$ and measurement error $M$ is required for the model to match the data perfectly along these two stylized facts. The results are $Q = 0.29$ and $M = 0.55$. As discussion above indicates, the empirical consumption response of 23 cents for each Euro implies that, for the PIH to be consistent with this fact, that income shocks are largely driven by transitory shocks (since permanent shocks imply a one-for-one consumption response). As discussed above, the size of measurement error plays essentially no role for the consumption regression coefficient in the model. Conditional on a value for $Q$ determined from the consumption data, the empirical wealth response then determines the required degree of measurement error.

---

Given equations (5)-(6) we can simply solve for $Q, M$ given the observed $\beta_{2c}^2, \beta_{2w}^2$ as

\[
Q = \frac{\beta_{2c}^2 - r \beta_{2w}^2}{1 - \beta_{2c}^2 + r \beta_{2w}^2}
\]

\[
M = 1 - \frac{2(1 + r) \beta_{2w}^2}{1 - \beta_{2c}^2 + r \beta_{2w}^2}
\]
With a choice of $Q = 0.29$ and $M = 0.55$ the PIH model matches the consumption and financial wealth response to labor income shocks over a two year horizon by construction. Thus this fact cannot be interpreted as a success of the model per se. However we would like to point out that while it is hard quantify the amount of measurement error of income in the data, the required value of income persistence $Q = 0.29$ is not implausible. With the panel dimension for labor income one can estimate $Q$ directly from the data, conditional on our assumption about the particular form of the income process. Jappelli and Pistaferri (2006) do exactly this for the Italian SHIW data and find $Q = 0.34$, somewhat higher, but in the range of the value required for the PIH to work well in a quantitative sense.\footnote{In appendix B we show that, if the first stage regression that controls for household observables fails to perfectly purge predicted income changes from the data, then the PIH predicts a lower regression coefficient for consumption than the one derived here.} In the next section we investigate whether an extension of the current model that includes a precautionary savings motive and thus implies that consumption responds to permanent income shocks less than one for one (see Carroll, 2009) can rationalize the observed consumption response of 23 cents with a persistence $Q$ even more in line with the empirical estimates by Jappelli and Pistaferri.

Before turning to the precautionary saving model we now more fully exploit the unique panel dimension of the Italian data to evaluate the predictions of the PIH for income shocks over longer time horizons, that is, for increasing $N$. An increase in $N$ means that more permanent shocks have accumulated, and that consumption should respond more strongly to a given income change. In table 4 we summarize how the model-implied consumption regression coefficients vary with $N$. Since the sample size falls significantly as $N$ increases, we restrict attention to $N \leq 6$. The model results are derived under the assumptions that $r = 2\%$, $M = 0.55$ and $Q = 0.29$, the values needed for the model to exactly match the data for $N = 2$ and wealth being interpreted as financial wealth.

<table>
<thead>
<tr>
<th>$N$</th>
<th>$\beta_c^N$</th>
<th>$\beta_w^N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.23</td>
<td>0.17</td>
</tr>
<tr>
<td>4</td>
<td>0.37</td>
<td>0.14</td>
</tr>
<tr>
<td>6</td>
<td>0.47</td>
<td>0.12</td>
</tr>
</tbody>
</table>

We observe that, as discussed earlier, the model predicts the expected increase in the consumption coefficients and the decline in the wealth coefficients with the time horizon.
For consumption the data suggests the same qualitative pattern, although the increase in the data is somewhat smaller than implied by the model. Furthermore, the pattern of the financial wealth response to income shocks is also broadly consistent with the data, displaying a decline in the wealth response as the time horizon increases from $N = 2$ years to $N = 6$ years. Note that the findings for $N = 4, 6$ provide a true test for the model as all model parameters have been chosen only with the data for $N = 2$ serving as targets.

To summarize, we conclude that the simple PIH model is successful in reproducing the empirically observed dynamic consumption and financial wealth response to income shocks of various durations. There are, however, two remaining empirical observations that this model has trouble in rationalizing. First, the required degree of persistence of income shocks seems at the high end of what the data suggests. Therefore in the next section we evaluate whether introducing a precautionary savings motive into the model allows the model to match the facts with the empirically estimated $Q = 0.34$ by Jappelli and Pistaferri (2006).

Second, the PIH cannot match the observed income-wealth correlations if wealth is interpreted more broadly to include real estate wealth (and business wealth), an interpretation that is mandated by a model that includes real estate explicitly (see appendix C). We therefore, in section 5 investigate further what could explain the observed large positive correlation between labor income shocks and real estate and business wealth.

### 4.2 A Precautionary Saving Model with CRRA Utility

The permanent income model abstracts from borrowing constraints and prudence in the utility function (by assuming that $u'''(c) = 0$). We now add these model elements that are well-known to give rise to precautionary savings behavior and thus may have the potential to reduce, quantitatively, the response of consumption to income shocks.

We envision a single household with monetary utility function $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ that faces the tight borrowing constraint $w_{t+1} \geq 0$. In addition in some versions of the model we cast the household in a life-cycle context. Households live for 61 periods (from age 20 to 80 in real time). Prior to retirement at age 65, income of a household of age $t$ is given by $y_t = \tilde{y}_t y_t$ where the stochastic part of income $\tilde{y}$, in logs, is specified as a random walk plus a transitory shock.

\[
\log(\tilde{y}_t) = z_t + \varepsilon_t \\
z_t = z_{t-1} + \eta_t
\]

with $\varepsilon_t \sim N(-\frac{\sigma^2_{\varepsilon}}{2}, \sigma^2_{\varepsilon})$ and $\eta_t \sim N(-\frac{\sigma^2_{\eta}}{2}, \sigma^2_{\eta})$. The means of the innovations are chosen such
that $E(\bar{y}) = 1$. After retirement households receive a constant fraction of their last pre-retirement permanent income $\bar{y}_t \exp(z_t)$ as pension. The income component $\bar{y}_t$ denotes the deterministic mean income at age $t$ and follows the typical hump observed in the data.\footnote{In the infinite horizon of the model we set $\bar{y}_t = 1$ for all $t$.}

The purpose of this section is to evaluate the potential of precautionary savings models to deliver smaller consumption responses to income shocks. Rather than carrying out an explicit calibration of the model we select parameter values that are plausible (relative to the existing literature) and constitute a minimal deviation form the pure PIH discussed above. With this objective in mind we select a CRRA of $\sigma = 2$ and choose $\rho = r = 2\%$, where $\rho = \frac{1}{\beta} - 1$ is the time discount rate of households. Jappelli and Pistaferri (2006, table 3) estimate $\sigma^2_\varepsilon = 0.0794$ and $\sigma^2_\eta = 0.0267$. Households start their life with $w_0 = 0$ and $z_{-1} = 0$.

In table 6 we summarize the consumption and in table 7 the wealth response to a labor income shock over various time horizons, both in the data as well as in various models. The column labeled "PIH" is derived from the PIH model in which infinitely lived households face an income process of the form in (7), with variances of permanent and transitory shocks specified in the previous paragraph. For comparison we also include the results (labeled "Analytical") obtained with an income process specified in levels, in which case we have provided the analytical expression of the regression coefficients in the previous section. The fact that the results differ slightly from the previous section is due to the fact that Jappelli and Pistaferri estimate a $Q = \frac{0.0267}{0.0794} = 0.34$ instead of the $Q = 0.29$ we had "calibrated". We observe that for all practical purposes it does not matter for the regression coefficients in the PIH model whether the income process is specified in levels or in logs.

The column CRRA ($T = \infty$) refers to results from the precautionary saving model with infinite horizon, where we first solved the model for the optimal policy functions and then simulated the model for 45 periods\footnote{In this model consumption and wealth diverges to $\infty$ almost surely, therefore we cannot sample from the ergodic distribution, but rather simulate a large number of households for a short period of $T = 45$, starting always from the same initial condition $w_0 = 0$ and $\eta_{-1} = 0$.}, with the initial conditions specified in the previous paragraph, and finally ran exactly the same regressions on the model-generated data as we did for the real data. The last column shows results from the same procedure for the precautionary saving model with an explicit life cycle income profile.
We observe that, for consumption, the precautionary savings motive indeed reduces the consumption response to an income shock (compare the third and forth column). Since with finite horizon later in life transitory shocks are essentially permanent shocks as well, the precautionary model with finite horizon implies larger consumption responses than the corresponding precautionary model with infinite horizon. Both versions imply, as does the PIH model and the data, that the consumption response increases with $N$. In addition, the precautionary savings model’s predictions with $T < \infty$ are remarkably close to the data, in a quantitative sense, despite the fact that it was not calibrated to achieve these targets. The same is true, along the consumption dimension, for the basic version of the PIH, even if the empirical estimates by Jappelli and Pistaferri for the income shock variances are used.

Table 7 displays the corresponding wealth response. We observe that while the PIH is qualitatively consistent with the decline of the wealth response with an increase in the time horizon $N$ (which we already documented in table 5). The CRRA models, on the other hand, predict a substantial increase in the wealth response over longer time horizons, qualitatively (and of course, quantitatively) at odds with the data. As $N$ increases, the persistence of the income shock increases, an effect that ought to reduce the wealth response to income shocks with increasing $N$. On the other hand, in these models even permanent shocks do not fully translate into corresponding consumption movements. Part of the

---

Table 6: Consumption Response

<table>
<thead>
<tr>
<th>$N$</th>
<th>Data</th>
<th>PIH [Analytical] &amp; CRRA ($T = \infty$)</th>
<th>CRRA ($T &lt; \infty$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.23</td>
<td>0.25 [0.26]</td>
<td>0.13</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
<td>0.40 [0.41]</td>
<td>0.21</td>
</tr>
<tr>
<td>6</td>
<td>0.27</td>
<td>0.50 [0.51]</td>
<td>0.26</td>
</tr>
</tbody>
</table>

---

Table 7: Financial Wealth Response

<table>
<thead>
<tr>
<th>$N$</th>
<th>Data</th>
<th>PIH [Analytical] &amp; CRRA ($T = \infty$)</th>
<th>CRRA ($T &lt; \infty$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.17</td>
<td>0.38 [0.37]</td>
<td>0.57</td>
</tr>
<tr>
<td>4</td>
<td>0.07</td>
<td>0.30 [0.29]</td>
<td>0.82</td>
</tr>
<tr>
<td>6</td>
<td>0.09</td>
<td>0.26 [0.25]</td>
<td>1.14</td>
</tr>
</tbody>
</table>

---

19Quantitatively, the wealth response in the model is larger, for every value of $N$, than in the data. Adding the appropriate degree of measurement error in income into the model would bring the model better in line with the data along this dimension, as we already demonstrated in the previous section.
permanent income shocks are absorbed by wealth. Over longer periods more permanent shocks accumulate, so the correlation between income and wealth changes increases with the time horizon \( N \). In models where permanent income shocks translate into consumption one for one (as in the original PIH or the CARA model) wealth does not absorb any of the permanent shocks. Thus in these models the wealth response to income shocks increase with the time horizon as over longer time horizons the importance of permanent income shocks increase, relative to transitory shocks (see the previous section). The fact that even permanent shocks are partially insured in the CRRA model lets the wealth response to income shocks rise with \( N \), and also reduces the increase of the consumption response with increased \( N \), relative to the PIH model.

We conclude that while the CRRA model with its partial insurance against permanent shocks is helpful in reducing the overall consumption response to income shocks towards the levels observed in the data, it is qualitatively inconsistent with the dynamic response of financial wealth to income shocks. Therefore we conclude that, overall, the simple PIH describes the consumption and wealth response to income shocks to a better, and very reasonable degree, at least for households without real estate and business wealth. It remains to be explored, however, what drives the large observed co-movement between income shocks and real estate and business wealth observed in the data. We turn to this question next.

5 What Drives the Co-Movement between Income Shocks and Real Estate and Business Wealth?

5.1 Real Estate Wealth

In Italy real estate is the predominant form of wealth held by private households. The median wealth household in 2006 owned about 140,000 Euro worth of real estate, relative to financial wealth of about 7000 Euro. As a point of comparison, median annual household income amounted to about 26,000 Euro. Mortgage debt, on the other hand is not very prevalent. Despite substantial increases in the last years the mortgagee debt to disposable

\footnote{Carroll (2009) demonstrates this result analytically and computationally in a model that is essentially identical to the one we use here.}

\footnote{Table 6 shows that the difference in the consumption response between \( N = 2 \) and \( N = 6 \) is 0.25 in the PIH model, but only 0.13-0.18 in the CRRA model that implies partial self-insurance against permanent shocks.}

\footnote{The degree of prudence, as measured by \( \sigma \), impacts the results quantitatively, but not qualitatively. Ceteris paribus, the larger is \( \sigma \) (the more prudent households are), the smaller is the consumption response and the larger is the wealth response. Furthermore, the consumption response increases more slowly with \( N \) the larger is \( \sigma \), and the wealth response increases more rapidly.}
income ratio is a mere 20%. Consequently, real estate is by far the most important component of total net worth of the median household. With 69% the home ownership rate is high and comparable to that of the U.S. As a further indicator of the importance of real estate wealth in a typical households’ portfolio, note that about 30% of all Italian households own more than one property, with the average number of properties being owned equal to 1.44 (the median number is 1, though). It is therefore not entirely surprising that adjustments in the real value of real estate may play an important role in a households’ response to an income shock.

The total net value of real estate owned by a household is given by the sum of the current market values of all properties owned net of the value of all outstanding mortgages, i.e.

\[ e = \sum_{i=1}^{\#N} p_i - m \]

where \( p_i \) is the price of owned property \( i \), \( \#N \) the number of properties owned and \( m \) total outstanding mortgage debt. Changes in the real value of owned real estate could then be due to a) house price changes of continuously owned properties, b) net new purchases (net changes in newly acquired minus sold properties) c) adjustments in value of mortgages (and thus equity shares) in owned properties.

Thus we can express the change in the value of real estate wealth as

\[
\Delta^2 e_{t+1} = \sum_{i=1}^{N^*} \Delta^2 p_{it} + \left( \sum_{i=1}^{N^{\text{new}}} p_{it} - \sum_{i=1}^{N^{\text{old}}} p_{it-2} \right) - \Delta^2 m_t \\
\equiv \Delta P + \Delta Q - \Delta m
\]

where \( N^* \) is the number of continuously owned properties between period \( t - 2 \) and \( t \), \( N^{\text{new}} \) is the number of newly purchased properties and \( N^{\text{old}} \) the number of sold properties between period \( t - 2 \) and \( t \).

Since we have detailed information about the self-assessed market value of each property a household owns, the year in which it was bought and the current use (primary residence, vacation home, rental property etc.) we can in principle construct all three components of changes in real estate wealth, \( \Delta P, \Delta Q, \Delta m \).

To obtain a first sense of the relative importance of the three components we now split the sample into three subsamples. The first is the subsample of households that do not adjust their real estate position (i.e. that have \( \Delta Q = 0 \)). The second subsample consists of all households in the first subsample that own some real estate (i.e. have \( e > 0 \)).
subsample consists of all households that adjust their position of real estate.

In table 8 we summarize the regression results obtained for these different samples. First, we observe that for all samples mortgages do not co-vary significantly with income changes, indicating the minor importance of the $\Delta m$ channel. This result may have been anticipated because of the relative unimportance of mortgages in Italy, and the fact that prepayment of mortgages and taking out second mortgages is highly uncommon. In fact, to the extent that there is any correlation between income changes and changes in the value of outstanding mortgages, it goes into the wrong direction. The regression coefficient for $\Delta m$ is positive, suggesting that households with positive income changes increase the value of their outstanding mortgages, although the magnitude is small. This finding presents evidence against the view that income increases are used to purchase real estate with leverage, resulting in a more than one for one increase in the gross value of real estate, relative to the income change. See appendix C for the details.

<table>
<thead>
<tr>
<th>$\beta^{NonAdj}_{LAD}$ [4761]</th>
<th>$\Delta c_n$</th>
<th>$\Delta (a + e)$</th>
<th>$\Delta e^{re}$</th>
<th>$\Delta a$</th>
<th>$\Delta m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.2</td>
<td>78.2</td>
<td>11.5</td>
<td>15.8</td>
<td>0.5</td>
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<tr>
<td>(0.7)</td>
<td>(5.8)</td>
<td>(2.7)</td>
<td>(1.5)</td>
<td>(0.1)</td>
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<table>
<thead>
<tr>
<th>$\beta^{NonAdjPR}_{LAD}$ [1619]</th>
<th>$\Delta c_n$</th>
<th>$\Delta (a + e)$</th>
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<tr>
<td>17.1</td>
<td>104</td>
<td>52.1</td>
<td>20.7</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>(1.2)</td>
<td>(12.5)</td>
<td>(10.4)</td>
<td>(2.8)</td>
<td>(0.2)</td>
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</table>

<table>
<thead>
<tr>
<th>$\beta^{Adj}_{LAD}[7875]$</th>
<th>$\Delta c_n$</th>
<th>$\Delta (a + e)$</th>
<th>$\Delta e^{re}$</th>
<th>$\Delta a$</th>
<th>$\Delta m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.9</td>
<td>191</td>
<td>75.7</td>
<td>17.9</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>(0.3)</td>
<td>(4.2)</td>
<td>(4.4)</td>
<td>(0.8)</td>
<td>(0.01)</td>
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</tr>
</tbody>
</table>

Second, the overall magnitude of the wealth-income shock correlation is significantly smaller for nonadjusters than for adjusters when wealth is measured as including all sources of wealth. Plausibly, nonadjusters rely slightly more strongly on the adjustment of financial wealth. Third, one could conclude that for nonadjusters the co-movement between changes in real estate wealth and income shocks is low (11.5 cents for each Euro, yet highly significant). But this number is mainly driven by the fact that among the nonadjusters a large share does not possess real estate wealth. In fact, as the second row of the table shows, conditional on having real estate wealth the co-movement between real estate wealth and income shocks is very substantial, albeit not quite as large as for the adjusters. Mechanically, this must mean that for nonadjusting households with positive real estate wealth there is a strong positive correlation between reported income changes and reported real
estate price changes of the continually owned properties. This correlation could possibly stem from a strong positive correlation of local housing and local labor markets\footnote{For the U.S. Davidoff (2006) documents a strong positive correlation between income growth and house price growth at the local level over five year horizons. He merges panel data on wages by region (MSA) and industry (2 digit SIC) from the BLS with regional (MSA) house price data from OFHEO and estimates an average (over MSA-industry pairs) correlation between house price and income growth of 0.29. The highest correlation (0.64) is obtained for households working in the amusement industry in the Orlando area. Davis and Ortalo-Magné (2011) find a strong positive correlation of 0.81 of mean (standardized) wage levels and rents in a year 2000 cross-section of MSA’s.}

5.2 The Role of Self-Employment

Households in which the household head is self-employed constitute a significant share of households with the largest changes in labor income, suggesting that these households face substantially more income risk. It is therefore instructive to investigate whether and to what extent the consumption and wealth response of this group differs from the overall sample. In table 9 we split the sample into self-employed households and their complement.

<table>
<thead>
<tr>
<th></th>
<th>$\Delta c_n$</th>
<th>$\Delta (a + e)$</th>
<th>$\Delta e^{re}$</th>
<th>$\Delta e^{bw}$</th>
<th>$\Delta a$</th>
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<tbody>
<tr>
<td>$\beta_{self}^{LAD}$ [2613]</td>
<td>5.7</td>
<td>148.2</td>
<td>20.9</td>
<td>17.8</td>
<td>15.8</td>
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<tr>
<td></td>
<td>(0.4)</td>
<td>(6.1)</td>
<td>(3.5)</td>
<td>(1.0)</td>
<td>(0.8)</td>
</tr>
<tr>
<td>$\beta_{emp}^{LAD}$ [10023]</td>
<td>20.5</td>
<td>117.2</td>
<td>44.8</td>
<td>9.3</td>
<td>18.5</td>
</tr>
<tr>
<td></td>
<td>(0.4)</td>
<td>(5.9)</td>
<td>(3.2)</td>
<td>(1.3)</td>
<td>(1.2)</td>
</tr>
</tbody>
</table>

We observe that self-employed households have significantly lower consumption and higher wealth responses than other households, where the wealth response is mainly driven by the strong positive co-movement between labor income and business wealth. As with real estate wealth, one plausible explanation is that shocks to labor income are positively correlated with the value of the business wealth the entrepreneur owns, without necessarily suggesting an active adjustment of the household in response to labor income shocks.

6 Conclusion

How do households respond to an income shock? In this paper we presented evidence that Italian households surveyed in the SHIW adjust nondurable consumption by 23 cents for each Euro and financial wealth by 17 cents. Evidence from the PSID is quantitatively similar. These observations are consistent with the permanent income hypothesis if most income shocks are transitory in nature. We also documented a strong positive correlation
between labor income shocks and adjustments in the value of real estate and business wealth. These findings suggest that shocks other than labor income shocks are important in shaping household economic decisions, and that these shocks (such as shocks to local house prices and businesses) might be strongly correlated with labor income shocks faced by households. Future research should address in more detail the forces behind this large correlation and could provide a uniform consumption-savings model that incorporates these shocks, and endogenizes the housing adjustment and business ownership decision.

References


A Variable definitions and panel dimension

Nondurable consumption $c_{nt}$ is defined as all household expenditures during a year, minus expenditures on transportation equipment (cars, bikes etc.), valuables (such as art, jewelry, antiques), household equipment (such as furniture, rugs, TV’s, cell phones and other electronics), expenditure for home improvement, insurance premia and contribution to pension funds. It includes rent paid by renters and imputed rent of home owners on all properties that are not rented out. Imputed rent also appears as income from real assets in $r_{et}e_t$ on the right hand side of the budget constraint. Expenditures on durables $c_{dt}$ include expenditures for transportation equipment, valuables and household equipment, all as defined above.

Labor income $y_t$ is measured after taxes and includes fringe benefits received by employees and business income by entrepreneurs. Transfers $T_t$ include both transfer payments from the government (such as unemployment benefits) as well as gifts, loans and other transfers between private households.

Financial assets $a_{t+1}$ add bank deposits, stock and bond holdings and other direct holdings of financial assets (including assets held in private pension funds), net of outstanding debt. It does not include the value of entitlements to government pension payments. The net income from financial assets (interest payments, dividends etc.) forms financial income $r_{at}a_t$. Finally, real assets $e_{t+1}$ include the value of real estate property, the value of valuables (as defined above) and the net value of ownership in private businesses and partnerships. Income from real assets, $r_{et}e_t$, consists mainly of rent (both actual and imputed) received from owned real estate.

Table A1 documents the total sample size by year and the extent of the panel dimension of the SHIW as every column reports the total number of households interviewed in that year and then divides them by their entry year in the SHIW.
Table A1. SHIW sample size and panel dimension

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<td>Total sample</td>
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<td>8188</td>
<td>8088</td>
<td>8135</td>
<td>7147</td>
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<td>8011</td>
<td>8012</td>
<td>7768</td>
<td>7977</td>
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</table>

B Predictable Income Changes

To the extent that our first stage regression that conditions the data on observables such as age, education etc. has failed to capture all predictable movements in income, the empirical estimates may partially reflect the consumption response to predictable income changes.\(^{24}\) The PIH model of course implies that consumption should not respond to predictable changes in income at all. Denoting the predictable part of income by \(\bar{y}_t\) the model now implies, for an income process

\[
y_t = \bar{y}_t + z_t + \varepsilon_t + \gamma_t
\]

\[
z_t = z_{t-1} + \eta_t
\]

\(^{24}\)On the other hand, It is possible that some of the variation the first stage regression picks up may have been predicted by the econometrician, but not by the household itself.
the model solution

\[ \Delta y_t = \eta_t + \Delta \bar{y}_t + \Delta \varepsilon_t + \Delta \gamma_t \]
\[ \Delta c_t = \frac{r}{1 + r} \varepsilon_t + \eta_t \]
\[ \Delta a_{t+1} = \frac{\varepsilon_t}{1 + r} - \frac{1}{1 + r} \sum_{s=1}^{\infty} \frac{\Delta \bar{y}_{t+s}}{(1 + r)^{s-1}}. \]

N-period changes are therefore given by

\[ \Delta^N c_t = \sum_{\tau=t-N+1}^{t} \left( \frac{r \varepsilon_\tau}{1 + r} + \eta_\tau \right) \]
\[ \Delta^N y_t = \sum_{\tau=t-N+1}^{t} \eta_\tau + \Delta^N \bar{y}_t + \Delta^N \varepsilon_t + \Delta^N \gamma_t \]
\[ \Delta^N a_{t+1} = \sum_{\tau=t-N+1}^{t} \frac{\varepsilon_\tau}{1 + r} - \frac{1}{1 + r} \sum_{s=1}^{\infty} \frac{\Delta^N \bar{y}_{t+s}}{(1 + r)^{s-1}} \]

and the regression coefficients implied by the model now read as

\[ \beta^N_c = \frac{N \sigma^2_\eta + r \sigma^2_\varepsilon / (1 + r)}{N \sigma^2_\eta + 2 (\sigma^2_\varepsilon + \sigma^2_\gamma) + \text{Var}(\Delta^N \bar{y}_t)} \]
\[ \beta^N_w = -\sum_{s=1}^{\infty} \frac{\text{Cov}(\Delta^N \bar{y}_t, \Delta^N \bar{y}_{t+s})}{(1 + r)^s} + \frac{\sigma^2_\varepsilon / (1 + r)}{N \sigma^2_\eta + 2 (\sigma^2_\varepsilon + \sigma^2_\gamma) + \text{Var}(\Delta^N \bar{y}_t)} \]

Thus the consumption response to income shocks goes down in presence of predicted income changes, the extent to which is determined by how large the cross-sectional variance in the N-period change in the predictable component of income is, relative to the variance of the permanent and transitory income shocks. Note that the wealth response to income changes now depends also crucially on the covariance of current and future predicted income changes.

### C The CRRA Model: Some Intuition

In this section we explore further the reasons behind the quantitative findings in the CRRA model that, as the horizon \( N \) between observations increases, both the consumption response and the wealth response to a given income shock increases. In the pure permanent income model consumption responds to a permanent income shock one for one, and wealth does
not respond at all. Suppose the precautionary savings motive embedded in the CRRA model with portentially binding borrowing constraints reduces the consumption response to a permanent shock from 1 to $\kappa$ and leave the consumption response to a transitory shock unchanged at $\frac{r}{1+r}$. Wealth absorbs the remainder of the shocks. Also suppose that households start with zero wealth $w_0 = 0$ and $\eta_{-1} = 0$ as initial conditions and income follows a random walk plus transitory shock in levels. Note that we by no means suggest that the assumed decision rule is indeed the correct one implied by the model. Rather we want to provide an intuitive argument why a model that leads to a consumption response of less than 100% to a permanent income shock (due to, say, a precautionary savings motive) generates regression coefficients $\beta^N_w$ that are increasing in $N$.

Under the assumed decision rule and initial conditions we have

$$w_{t+1} = (1 - \kappa) \sum_{\tau=0}^{t} \sum_{j=0}^{\tau} (1 + r)^j \eta_{t-\tau} + \frac{1}{1+r} \sum_{\tau=0}^{t} \varepsilon_{\tau}$$

$$c_t = \sum_{\tau=0}^{t} \left( \kappa \eta_{\tau} + \frac{r \varepsilon_{\tau}}{1+r} \right)$$

$$y_t = \sum_{\tau=0}^{t} \eta_{\tau} + \varepsilon_t$$

so that

$$\Delta y_t = \eta_t + \varepsilon_t - \varepsilon_{t-1}$$

$$\Delta c_t = \kappa \eta_t + \frac{r \varepsilon_t}{1+r}$$

$$\Delta w_{t+1} = (1 - \kappa) \sum_{\tau=0}^{t} \eta_{\tau}(1 + r)^{t-\tau} + \frac{\varepsilon_t}{1+r}$$

Taking $N$-th differences and computing variances and co-variances yields

$$\beta^N_c = \frac{\kappa N Q + \frac{r}{1+r}}{N Q + 2}$$

$$\beta^N_w = \frac{(1 - \kappa) \Psi(N) Q + \frac{1}{1+r}}{N Q + 2}$$

where

$$\Psi(N) = \sum_{j=0}^{N-1} (1 + r)^j (N - j)$$

37
We note that

\[
\text{sgn}(\beta_{c}^{N+1} - \beta_{c}^{N}) = 2\kappa - \frac{r}{1 + r}
\]

\[
\text{sgn}(\beta_{w}^{N+1} - \beta_{w}^{N}) = \text{sgn}\left[\frac{(1 - \kappa)\Psi(N + 1)Q + \frac{1}{1 + r} - (1 - \kappa)\Psi(N)Q + \frac{1}{1 + r}}{(N + 1)Q + 2} - \frac{1}{NQ + 2}\right]
\]

and thus as long as \(\kappa > \frac{r}{2(1 + r)}\) but \(\kappa\) not too large (that is, consumption response sufficiently weakly to permanent shocks) we have that both \(\beta_{c}^{N}\) and \(\beta_{w}^{N}\) are increasing in \(N\).

D Housing in the Standard Incomplete Markets Model

We now introduce housing explicitly into the standard incomplete markets model. We first model the housing choice of households without any frictions in the adjustment of real estate position and no explicit borrowing constraints.\(^{25}\) Also, households have access to a competitive rental market where housing services \(s_{t}\) can be rented for a rental price \(R_{t}\) per unit of house. Households buy real estate \(h_{t+1}\) at price per unit of \(p_{t}\), as well as nondurable consumption \(c_{nt}\) and financial assets \(a_{t+1}\). Houses depreciate at rate \(\delta\). The household decision problem is then given by

\[
\max_{\{c_{nt}, s_{t}, a_{t+1}, h_{t+1}\}} \sum_{t} \beta^{t}v(c_{nt}, s_{t})
\]

\[
c_{nt} + a_{t+1} + R_{t}s_{t} + p_{t}h_{t+1} = y_{t} + (1 + r_{t})a_{t} + p_{t}(1 - \delta)h_{t} + R_{t}h_{t}
\]

where \(v(c_{nt}, s_{t})\) gives the period utility from consuming nondurables \(c_{nt}\) and housing services \(s_{t}\).

D.1 Analysis

It is straightforward to show that this household problem can be solved in three stages. In the first stage the intratemporal consumption allocation problem between nondurables and housing services is solved

\[
\max_{c_{nt}, s_{t}} v(c_{nt}, s_{t})
\]

\[
c_{nt} + R_{t}s_{t} = c_{t}
\]

\(^{25}\)Of course an appropriate no-Ponzi condition has to be imposed to make the household problem have a solution.
where $c_t$ is the expenditure on housing services. The solution characterized by the two equations

$$\frac{v_s(cn_t, s_t)}{v cn(cn_t, s_t)} = R_t$$

$$cn_t + R_t s_t = c_t$$

Define the indirect utility function resulting from this maximization problem as

$$u(c_t; R_t) = v(cn_t(c_t, R_t), s_t(c_t, R_t))$$

This is the period utility function used in the main text.

In a second stage the household decides how to split its savings between financial and real assets. Without any frictions in the real estate market (or the financial asset market, for that matter) a simple no-arbitrage argument implies that the rental price and the price of real estate have to satisfy the condition.

$$R_{t+1} = p_t \left[ (1 + r_{t+1}) - \frac{p_{t+1}(1 - \delta)}{p_t} \right]$$

Under this condition one can consolidate both assets into one

$$w_{t+1} = a_{t+1} + p_t h_{t+1}.$$

Exploiting the outcome of steps i) and ii) the intertemporal household problem then reads as

$$\max_{\{c_t, w_{t+1}\}} E_0 \sum_t \beta^t u(c_t; R_t)$$

$$c_t + w_{t+1} = y_t + (1 + r_t) w_t$$

where consumption expenditures and wealth are measured as

$$c_t = cn_t + R_t s_t$$

$$w_{t+1} = a_{t+1} + p_t h_{t+1}$$

$$= a_{t+1} + e_{t+1}.$$

As long as $c_t$ and $w_t$ are measured empirically consistent with the theory, the analysis can proceed as in the main text, without explicit consideration of the households’ housing
D.2 Adding Financing Constraints

Suppose the household can only finance a fraction $1 - \gamma$ of the value of real estate purchased in the current period,

$$a_{t+1} \geq -(1 - \gamma)p_t h_{t+1}.$$  

The effect of such a constraint on the dynamics of the stock of real estate was studied, among others, by Fernandez-Villaverde and Krueger (2011), Campbell and Hercowitz (2009) and Aaronson et al. (2011). The presence of such a constraint may significantly alter the response of housing wealth to a change in income. Suppose that households find it optimal to be at the constraint in period $t$, then $a_{t+1} = -(1 - \gamma)p_t h_{t+1}$. Substituting this into the budget constraint (8) yields

$$c_t + \gamma p_t h_{t+1} = y_t + (1 + r_t) a_t + p_t (1 - \delta) h_t + R_t h_t.$$  

Therefore if households are constrained in periods $t - 1$ and $t$ we have

$$\frac{\Delta c_t}{\Delta y_t} + \frac{\gamma \Delta p_t h_{t+1}}{\Delta y_t} = 1.$$  

It is straightforward to observe $\frac{\Delta p_t h_{t+1}}{\Delta y_t} > 1$, that is expenditures on nondurables and net new housing can exceed the income change since households can leverage home purchases. But also note that this implies that

$$\frac{\Delta a_{t+1}}{\Delta y_t} = -(1 - \gamma) \frac{\Delta p_t h_{t+1}}{\Delta y_t}$$  

and thus one would expect large adjustments in the value of mortgages (or other financial debt), too. This is not what the empirical analysis in section 5 of the main text reveals.

D.3 Adding Prohibitive Transaction Costs

Now imagine a household with current size of housing stock $h$ that will never move, perhaps because of prohibitively high transaction costs. Also assume that the households lives in her own home, and does not rent out part of the home. Furthermore assume that the depreciation rate on houses is $\delta = 0$. Let the service flow from the owner-occupied house $h$ be given by $s = \phi(h)$, where $\phi$ is an arbitrary function. Then the household problem
(absent financing constraints) reads as

$$\max_{\{cn_t, a_{t+1}\}} E_0 \sum_t \beta^t v(cn_t, s)$$

$$cn_t + a_{t+1} + p_t(h_{t+1} - h_t) = y_t + (1 + r_t)a_t$$

and since by assumption $h_{t+1} = h_t$, the budget constraint reads as

$$cn_t + a_{t+1} = y_t + (1 + r_t)a_t.$$ 

Evidently, the household does not care at all about changes in house prices. As long as the utility function is additively separable in $cn$ and $s$ or satisfies

$$v(cn_t, s) = g(s)u(cn) = g(\phi(h))u(cn),$$

since $h$ is constant by assumption the presence of housing services simply represents an affine transformation of the period utility function that leaves nondurable consumption choice behavior unaffected. Thus under these assumption we can proceed with our analysis of the PIH or the more elaborate precautionary saving model as if housing wealth and services are not present in the model.

With a nonzero depreciation rate $\delta > 0$, house price changes affect measured disposable labor income $\tilde{y}_t = y_t - p_t \delta h$, but the model-implied map between the adjusted income measure and nondurable consumption and financial wealth remains unaffected.