Banks as Coordinators of Economic Growth and Stability: Microfoundation for Macroeconomy with Externality

Kenichi Ueda *
International Monetary Fund
email: kueda@imf.org

Abstract

Competition among banks promotes growth and stability for an economy with production externality. Following Arrow and Debreu (1954), I formulate a standard growth model with externality—a two-period version of Romer (1986)—as a game among consumers, firms, and intermediaries. The Walrasian equilibrium, with an auctioneer, does not achieve the social optimum. Without an auctioneer or intermediaries, I show that no Nash equilibrium exists. With several banks strategically intermediating capital, a Nash equilibrium emerges with a realistic institution, i.e., an interbank market with a negotiation process in the loan market. The equilibrium outcome is uniquely determined and socially optimal.

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I. INTRODUCTION

I formally identify an essential role that banks play: A strategically competitive banking sector serves as a decentralized mechanism to internalize positive production externality among firms, thereby facilitating economic growth. To clarify this role, I shut down banks’ roles that have been considered in the banking literature. The model does not assume any exogenous stochastic shocks, informational problems, illiquid projects, and transaction costs.\(^2\)

The strategically competitive banking sector also brings stability in an economy with production externality by supporting a Nash equilibrium. Without any financial intermediaries, I show that an economy with production externality, which often appears in the economic growth literature, faces serious instability because a Nash equilibrium does not exist. To support an equilibrium, I further find that an additional institutional setup among banks is necessary: an interbank market with a negotiation process in the loan market. This institutional setup can be viewed as an optimal mechanism to internalize the externality.

My model is based on a canonical growth model with externality, essentially the same as in Romer (1986).\(^3\) In his model, investment of a firm is assumed to raise marginal products of other firms. Because of this Marshallian externality, the competitive equilibrium is not Pareto optimal: Investment is lower and the growth is slower. This result has been supporting a case for subsidies (e.g., transportation and R&D) or patents for firms to increase their investments.

In Romer (1986) and many other growth papers, the capital market is assumed to be competitive in the Arrow-Debreu sense: Financial activity is conducted only by a security market. But, what happens if banks intermediate the capital market?

I first formulate a two-period version of the Romer growth model in the spirit of Arrow and Debreu (1954): An economy is a game among consumers, firms, and an auctioneer. The auctioneer, as an abstraction of a security market, intermediates the financial transaction and converts savings to capital. Not surprisingly, the equilibrium allocation is the same as in Romer (1986): The Walrasian equilibrium is not Pareto optimal.

I then modify the model so that banks clear the capital market. An economy becomes a game among consumers, firms, and banks. Banks strategically compete with each other in deposit and loan markets. I find that, with an additional institutional setup, a Nash equilibrium exists, and it is Pareto optimal.

Banks, in this paper, are different from the Walrasian auctioneer in Arrow and Debreu

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\(^2\)The banking literature, so far, has explained banks’ advantage over markets mainly as mitigation of informational problems and economization of transaction costs. See, for example, a textbook by Freixas and Rochet (1997).

\(^3\)The growth process with externality is originally considered by Shell (1966).
While the auctioneer offers only price, a bank can post a contract that specifies both price and quantity (e.g., loan rate and amount). With this more realistic contract space, every bank has an incentive to become a monopoly lender, because, if a bank becomes a monopoly lender, it can tailor the loan contracts to extract all the rents from all firms, including any external effects. Knowing this, banks will compete aggressively for deposits by driving the deposit rate up to the return that a monopoly lender would obtain. In the equilibrium, many banks and firms operate at zero profit under the interest rate that equals the socially optimal level. Also, in the equilibrium, savings and loan amounts become equal to the socially optimal levels.

There is a caveat, however. To be clear, the other allocation cannot be an equilibrium: The interest rate below the monopoly lender’s return would be upset by a rival bank, and the interest rate above the monopoly lender’s rate cannot be technically feasible to prevail. Yet, there is a profitable deviation at the identified equilibrium candidate. The private marginal return is lower than the social marginal return so that a bank-firm pair would want to invest less than the socially optimal amount and share extra profits by free riding on investments of other firms. As a consequence, no Nash equilibrium exists in an economy in which an auctioneer is simply replaced by banks.

The problem lies in the discontinuity of banks’ profit function. Banks are willing to bid the deposit rate up to the monopolist’s loan rate that internalizes externality. But, at this socially optimal interest rate, banks suddenly have to worry about their fund positions being too large and thereby want to limit deposit amounts. Therefore, the only remedy to support an equilibrium in a decentralized economy is to introduce an institutional setup that allows banks’ loan market behaviors to be somewhat independent from their collected deposits (a weak link between sources and uses of funds).

Apparently, introducing the interbank market is necessary to break the constraint that forces each bank’s loan to be strongly tied to its collected deposits. In addition, some sort of price adjustment mechanism is necessary for the interbank market to clear. I propose a simple, realistic mechanism: Banks are allowed to have a free-recontracting opportunity in the loan market so that they can adjust quotes on loan terms. For example, if there were two sessions in a day (e.g., morning and afternoon), banks could freely change their loan offers once before the settlement at the end of the day. This mechanism is sufficient to support the identified Nash equilibrium. Note that potential instability may still remain if a more refined equilibrium than a Nash equilibrium is required. However, the instability is not created by banks but deeply rooted in the economy.4

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4In game-theoretic terms, this economy is described as a discontinuous game in which payoff functions are too discontinuous to support a Nash equilibrium. In this literature (e.g., Reny, 1999), finding a condition to support a Nash equilibrium is the main theme, not refining it (e.g., subgame perfection and trembling-hand perfection).
In one strand of the literature on linkages between a macroeconomy and its financial system, many studies argue that banks are superior to a market allocation by their more active involvement in investments, especially in the phase of economic development. They are, however, mostly based on historical descriptions (e.g., Gerschenkron, 1962; Cameron and others, 1967; Aoki and Patrick, 1994; Allen and Gale, 2000; and Guinnane, 2002). Also, there are formal modeling attempts on finance and growth (e.g., Greenwood and Jovanovic, 1990; Bencivenga and Smith, 1991; and Greenwood and Smith, 1997), but banks’ specific roles in economic growth with production externality have not been clearly delineated or argued, at best, in a partial-equilibrium setting (e.g., Da Rin and Hellmann, 2002).

In another strand of the literature on finance-macroeconomy linkages, instability is the key issue. Instability is associated with riskiness of loans in banking theories, while instability means amplification of cycles by financial frictions in macroeconomic models. In contrast, this paper deals with more fundamental instability, i.e., possible nonexistence of an equilibrium. In the banking literature, several studies consider bank competition as the primary suspect that brings instability (e.g., Keeley, 1990), though some question this speculation (e.g., Boyd and De Nicolo, 2005). Many of these theories are, however, based on a partial-equilibrium setting, or with little connection to formal macroeconomic theories, and thus difficult to apply to growth and stability issues at the macroeconomic level. Moreover, any of these theories assume some frictions: private information, limited liability, or transaction costs. Also, in macroeconomic models with financial amplification (e.g., Bernanke and Gertler, 1989; and Kiyotaki and Moore, 1997), bank behaviors are mostly hidden in assumptions on financial frictions. Again, as this paper does not rely on any of those frictions, this paper brings a new perspective on the debate on bank competition and stability.

From a technical point of view, this paper can be regarded as an extension of the literature on strategic intermediation to a general equilibrium growth model with production. This literature has attempted to replace the Walrasian auctioneer with strategic firms or middlemen. Townsend (1983), Stahl II (1988), and Yannelle (1998) study the strategic

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This paper suggests that a slight modification of the strategy space can support a reasonable equilibrium.

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5Chernow (1990) describes a prototypical example:

During the pre-1913 Baronial Age of Pierpont Morgan, bankers were masters of the economy, or “lords of creation.” ... They financed canals and railroads, steel mills and shipping lines, supplying the capital for a nascent industrial society. ... As the major intermediaries between users and providers of capital, they oversaw massive industrial development. ... [They] acquired increasing control over [firms].

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6A notable exception includes a general equilibrium study by Allen and Gale (2004a). See recent reviews of this banking literature by Allen and Gale (2004b) and by Beck and others (2010).
competition of middlemen in a frictionless economy.\textsuperscript{7} Their common concern is whether strategic intermediaries achieve the Walrasian equilibrium. Results are mixed. Townsend (1983) shows positive results in an exchange economy.\textsuperscript{8} In a partial equilibrium framework, given traditional demand and supply functions, Stahl II (1988) shows mixed results that depend on specification of the game, and Yannelle (1998) reports a negative result, i.e., the allocation is inefficient.

In Section II, I describe the general model setup for an economy with intermediaries. In Section III, I further develop the model in detail to describe the strategically intermediated economy with free recontracting opportunity. I then show the existence of an equilibrium and the uniqueness of the equilibrium outcome. In Section IV, I explain nonexistence of an equilibrium in an economy without intermediaries. This case is isomorphic to an economy with intermediaries but without further institutional setup, such as free recontracting opportunity. Section V concludes.

\section{Model Setup}

\subsection{Demography, Technology, and Preference}

The economy is populated by consumers $i \in \{1, \cdots, I\}$, firms $j \in \{1, \cdots, J\}$, and intermediaries $h \in \{1, \cdots, H\}$. While consumers live for two periods, firms and intermediaries are active only in the first period.\textsuperscript{9} The number of intermediaries is assumed, without loss of generality, to be smaller than the number of firms. A firm borrows capital from consumers at the beginning of the first period, invests it in the production process, and returns outputs, equivalent to the sum of capital and interests, to consumers at the end of the first period.

The production technology is almost the same as in Romer (1986), a standard growth model with positive production externality. I write firm $j$’s capital as $k_j \in \mathbb{R}_+$ and let $\Psi$ denote the set of active firms: firms that invest positive amounts of capital. I also define the

\textsuperscript{7}Townsend (1978) addresses a similar issue, but in an economy with transaction costs, and points out that intermediaries emerge as they economize transaction costs. Also, see Yannelle (1997) for an analysis with transaction costs associated with private information.

\textsuperscript{8}Using the same structure as Townsend (1983), Acemoglu and Zilibotti (1997) show that introducing banks in their model of gradually expanding contingent claim markets does not make any difference from the market-based allocation, which is not the first best due to incomplete markets assumed in their model. The second of two models described in Greenwood and Smith (1997) is a similar general equilibrium model on a link between intermediaries and growth, but not particularly suitable to explain a link between banks and growth, as it is a game of market formation for many specialized intermediary goods by market makers, who act like a Walrasian auctioneer.

\textsuperscript{9}The setup can be easily extended to an infinite-period model.
set of active firms that is not the $j$-th firm as $-j \equiv \{l : l \in \Psi, l \neq j\}$. The number of firms in the active firms’ set $\Psi$ is denoted by $\#\Psi$. The population average capital for firm $j$ is defined as follows:

$$K_j \equiv \frac{1}{\#\Psi - 1} \sum_{l \in -j} k_l.$$  

(1)

Let $y_j$ denote output of firm $j$. Given the average capital $K_j$, firm $j$ produces its output from capital $k_j$ as much as

$$y_j = AK_j^\eta k_j^\alpha,$$  

(2)

where $A \in \mathbb{R}_+$ is the total factor productivity. Let $R_j$ denote the (weighted) average gross interest rate of capital that firm $j$ pays. The profit function of firm $j$, then, is defined as $\pi^f : \mathbb{R}_+^3 \rightarrow \mathbb{R}$, such that

$$\pi^f_j = \pi^f(k_j, K_j, R_j) \equiv AK_j^\eta k_j^\alpha - R_j k_j.$$

(3)

I focus on the case of $\eta = 1 - \alpha$. This is the case of constant returns to accumulated capital because, for the social planner who treats each firm equally, each firm’s capital level is viewed as the average, $k = k_j = K_j$, and the production function becomes linear in capital, $Ak$. Moreover, I assume $\alpha \in (1/2, 1)$; i.e., there always exists externality but still the own inputs matter more than the others’ inputs.

All consumers are identical in preferences. Let $\beta \in (0, 1)$ denote the discount rate; $c_{it} \in \mathbb{R}_+$ individual $i$’s consumption in period $t = 1, 2$; and $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ the period-utility function. To obtain internal solutions, the period-utility function $u$ is assumed to be twice continuously differentiable with the properties $u' > 0$ and $u'' < 0$ and to satisfy Inada conditions $\lim_{c_{it} \rightarrow 0} u'(c_{it}) = \infty$ and $\lim_{c_{it} \rightarrow \infty} u'(c_{it}) = 0$.

Given an initial wealth $m_i \in \mathbb{R}_{++}$, consumer $i$ maximizes her lifetime utility,

$$U(c_{i1}, c_{i2}) = u(c_{i1}) + \beta u(c_{i2}),$$  

(4)

subject to the budget constraints. In the first period, the budget constraint is

$$c_{i1} + s_i \leq m_i.$$  

(5)

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10I treat $K_j$ as the average, not aggregate, level of capital, because the aggregate capital level of a country should not affect its growth (Lucas, 1993). Also, in most parts of the paper, when there is no risk of confusion, I assume all firms are active; the number of all firms $J$ is used in place of the number of active firms $\#\Psi$.

11As Romer (1986) noted, this is the only case that delivers perpetual growth and, thus, has been the main interest of the growth literature. However, other cases will be discussed in Section III.E.
where \( s_i \) denotes savings. In the second period, it is

\[ c_{i2} \leq M_i, \tag{6} \]

where \( M_i \) is the wealth at the beginning of the second period.

Consumers own firms. Let \( \psi_{ij}^f \in (0, 1) \) be the ownership of the \( j \)-th firm by consumer \( i \). The sum of the ownership is one, \( \sum_{i=1}^{I} \psi_{ij}^f = 1 \), for every firm \( j \in J \). Similarly, consumers own financial intermediaries. Let \( \psi_{hi}^b \in (0, 1) \) be the ownership of the \( h \)-th intermediary by consumer \( i \), and the sum of the ownership is one, \( \sum_{i=1}^{I} \psi_{hi}^b = 1 \), for every intermediary \( h \in H \). Intermediary \( h \)'s profit is denoted by \( \pi_h^b \), whose detail is determined by a specific institutional setup explained later. Let \( w_i \) denote the total profit income of consumer \( i \). It is defined as

\[ w_i = \sum_{j=1}^{J} \psi_{ij}^f \pi_j^f + \sum_{h=1}^{H} \psi_{hi}^b \pi_h^b. \tag{7} \]

The wealth of consumer \( i \) in the second period, \( M_i \), consists of the gross return on savings and the profit income:

\[ M_i = r_i s_i + w_i, \tag{8} \]

where \( r_i \in \mathbb{R}_+ \) denotes the (weighted) average gross rate of return on savings for consumer \( i \).

There are two economy-wide resource constraints as follows:

(i) capital must be converted from savings,

\[ \sum_{j=1}^{J} k_j \leq \sum_{i=1}^{I} s_i; \quad \text{and} \tag{9} \]

(ii) consumption and savings are bounded by total output,

\[ \sum_{i=1}^{I} (c_i + s_i) \leq \sum_{j=1}^{J} y_j. \tag{10} \]

In addition, if an equilibrium is symmetric, the fixed-point condition (so called in the literature) applies to all active firms \( j \in \Psi \),

\[ k_j = K_j. \tag{11} \]

To focus on the allocation by the financial system, produced consumption goods \( y_j \) are assumed to be distributed to households as interest income and profit income. Households either consume or save the allocated consumption goods without selling or purchasing them in the product market. When households save consumption goods, they convert them into
capital goods. In other words, the capital market is assumed to be the only active market in the economy.\footnote{However, this setup is equivalent to assuming that a household receives labor income from firms and profit income from firms and banks and spends the total income to purchase goods from firms in the Walrasian product market.}

\section*{B. Pareto-Optimal Allocation and Walrasian Equilibrium}

To compare welfare among different institutional settings, a natural benchmark is the symmetric first-best solution (i.e., the socially optimal allocation with equal treatment of all households). The analysis is standard and omitted. The summary results are as follows: (i) a benevolent social planner internalizes externality by setting each firm’s capital at the same level, $k = k_j = K_j$, so that the planner will face the linear $Ak$ production technology; and (ii) the savings is determined, and the wealth is accumulated, under the social marginal return $A$.

Another benchmark is the Walrasian equilibrium, which does not bring the Pareto optimal allocation. In the Walrasian economy, there exists only one intermediary ($H = 1$) called an auctioneer. Its strategy is to pick an interest rate, common to both deposit and loan rates, to maximize its profit, i.e., the value of excess demand. In the equilibrium, the auctioneer matches demand and supply of capital, and its profit becomes zero. Online Appendix I shows more formal analysis. In essence, given the auctioneer’s offered interest rate, each firm determines its investment based on its private marginal return $\alpha AK_{j}^{1-\alpha}k_j^{\alpha-1}$ without taking into account the spillover effect on others. Because investments become symmetric $k_j = K_j$, the equilibrium interest rate is $\alpha A$, lower than the social marginal return $A$. As a consequence, wealth is accumulated less, and consumption growth is lower than in the Pareto-optimal allocation.

\section*{III. Economy with Strategic Intermediation}

I assume more than one bank is active $H \geq 2$, and I formulate the economy formally as a game among consumers, firms, and banks in the style of Arrow and Debreu (1954). Banks compete in both the deposit and loan markets. Although, traditionally, strategic competition has been analyzed in the form of either Bertrand or Cournot competition, this paper considers a more general concept of competition: competition in both price and quantity. In essence, banks have more sophisticated technology than an auctioneer: Banks can tailor deposit contracts for depositors and loan contracts for client firms by specifying both price and quantity.\footnote{The monopoly bank case ($H = 1$) is not worth considering. It is characterized by a monopoly bank, who internalizes externality in the loan market but offers a deposit rate lower than the loan rate to maximize its} I formulate the economy as a two-stage game, competition for deposits first and...
then for loans, but simultaneous opening of both the deposit and loan markets can be analyzed similarly.\textsuperscript{14}

A. Second Stage with a Monopoly Bank

A bank becomes a monopoly lender if it captures all savings in the deposit market, the first stage. Let $M \in \{1, 2, \cdots, H\}$ denote a monopoly lender. Its strategy is to post a loan contract to firm $j$. The contract consists of the loan rate $R_{Mj}^b \in \mathbb{R}_+$ and the amount $k_{Mj}^b \in \mathbb{R}_+$, and is denoted by $z_{Mj}^b \equiv (R_{Mj}^b, k_{Mj}^b)$. Loan rates and loan amounts must be nonnegative or “not specified,” abbreviated as N.S. The strategy set is then defined as $Z_M \equiv (\mathbb{R}_+ \cup \{\text{N.S.}\})^2J$.

Firm $j$’s strategy, when it faces an offer from a monopoly bank, is to choose a loan contract $z_{Mj}^f \equiv (R_{Mj}^f, k_{Mj}^f)$ from its strategy set $Z^f_M \equiv \mathbb{R}_+^2$. However, this strategy set is constrained by the bank’s offer $z_{Mj}^b$. The constrained choice set is written as $G_{M}^f(z_{Mj}^b)$. I assume that no borrowing, $k_{j} = 0$, is always in the choice set:

$$G_{M}^f(z_{Mj}^b) \equiv R_{Mj}^b \times \mathbb{R}_+ \quad \text{if the bank specifies } R_{Mj}^b \text{ only},$$

$$\equiv \mathbb{R}_+ \times (k_{Mj}^b \cup \{0\}) \quad \text{if the bank specifies } k_{Mj}^b \text{ only, and}$$

$$\equiv R_{Mj}^b \times (k_{Mj}^b \cup \{0\}) \quad \text{if bank } M \text{ specifies both } R_{Mj}^b \text{ and } k_{Mj}^b. \quad (12)$$

The firm’s choice set of the last case is either $(R_{Mj}^b, k_{Mj}^b)$ or $(R_{Mj}^b, 0)$; i.e., a firm has the choice to “accept” or “reject” the offer.\textsuperscript{15}

\textsuperscript{14}In the real world, there are several types of financial service providers, such as money lenders, wealthy financiers, and large finance departments in manufacturing firms. However, according to my model, financial activity must be clearly distinguished from firms’ manufacturing activity and consumers’ savings decisions. If an entity borrows and lends funds, I label it a bank. In this paper, financial decisions of a household are confined to deciding how much to save in available financial products. Similarly, a financial decision by a firm is to determine which financial contracts to take, among those available. This functional distinction is similar to what the standard microeconomic theory does between consumers and producers.

\textsuperscript{15}Note that the choice set is a possible space from which the strategy can be taken. As such, some strategies may never be taken. For example, the strategies without specifying the price (the second line of (12)) will never be taken as an equilibrium strategy: If the loan rate is not specified, a firm asks the bank for a zero loan rate. I include this strategy in the choice set for the sake of symmetric treatment of both price and quantity dimensions. On the other hand, this choice set may seem too restrictive in that it does not allow banks to offer a menu of financial contracts, e.g., a loan rate as a function of borrowing amount. A menu of contracts would be important in an economy with hidden types or hidden actions. However, with full information and with homogeneous firms (and households), extending a choice set to include a menu of contracts does not at all alter the analytical results of this paper.
The best response of a firm is defined as:

$$z^{fBR}_{Mj}(z^b_M, k^{f}_{M,j}) = \arg \max_{z^f_{Mj} \in G^f_M(z^b_M)} \pi^f(k^f_{Mj}, K^f_{Mj}, R^f_{Mj}).$$  \hspace{1cm} (13)$$

I write $z^b_M \equiv \{z^b_{Mj}\}_{j=1}^J$ to denote a vector of all banks’ offers and $z^{fBR}_{M} \equiv \{z^{fBR}_{Mj}\}_{j=1}^J$ to denote a vector of all firms’ best responses.\(^{16}\) The elements of $z^{fBR}_{Mj}$ are the borrowing rate $R^{fBR}_{Mj}(z^b_M, k^{f}_{M,j})$ and the borrowing amount $k^{fBR}_{Mj}(z^b_M, k^{f}_{M,j})$. Naturally, both are functions of all the banks’ offers $z^b_M$ and other firms’ strategies $k^{f}_{M,j}$.

The monopoly bank maximizes profit $\pi^b_{M}(z^b_M|z_D)$ by choosing the loan contracts they will offer to all firms, given an outcome of the deposit market $z_D$, namely, the deposit amount $s_{Mi}$ per consumer and its deposit rate $r_{Mi}$:

$$\max_{z^b_M \in Z^b_M} \pi^b_{M}(z^b_M|z_D) \equiv \sum_{j=1}^J R^{fBR}_{Mj}(z^b_M, k^{f}_{M,j})k^{fBR}_{Mj}(z^b_M, k^{f}_{M,j}) - \sum_{i=1}^I r_{Mi}s_{Mi},$$  \hspace{1cm} (14)$$

subject to the resource constraint, which is the bank balance sheet condition,

$$\sum_{j=1}^J k^{fBR}_{Mj}(z^b_M, k^{f}_{M,j}) \leq \sum_{i=1}^I s_{Mi}.$$  \hspace{1cm} (15)$$

**Definition 1.** Given a deposit market outcome $z_D$, the second stage with a monopoly bank is the game $\Gamma_M$, which consists of one bank and $J$ firms, their strategy sets, and their profit functions:

$$\Gamma_M(z_D) \equiv ((1, J), (Z^b_M, G^f_M), (\pi^b_{M}, \pi^f_{j})).$$  \hspace{1cm} (16)$$

An outcome is a set of loan rates and loan amounts, denoted without superscripts, $z_M = \{z_{Mj}\}_{j=1}^J$, where $z_{Mj} = (R_{Mj}, k_{Mj})$.

**Lemma 1.** The Nash equilibrium strategies of $\Gamma_M$ are

(i) the optimal decision of monopoly bank $h$, $z^{b*}_{M} = (R^{b*}_{Mj}, k^{b*}_{Mj})$, that satisfies

$$k^{b*}_{Mj} = \frac{\sum_{i=1}^I s_{Mi}}{J} \quad \text{and} \quad R^{b*}_{Mj} = A,$$  \hspace{1cm} (17)$$

(ii) the optimal decision by firms, which is to “accept” this offered contract (i.e., $z^{f*}_{Mj} = z^{b*}_{Mj}$).

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\(^{16}\)For a vector consisting of sequence $x_i$, I define the following notation: \((\{x_i\}_{i=1}^I) \equiv (x_1, x_2, \ldots, x_I)\). For $x_{ij}$, I use a similar notation: \((\{x_{ij}\}_{i,j=1}^{I,J}) \equiv (x_{11}, x_{12}, \ldots, x_{21}, x_{22}, \ldots, x_{IJ})\).
Therefore, the equilibrium outcome is nothing but the bank’s optimal strategy,

\[
Z^*_M = \left( \{ R^*_M, k^*_M \}_{j=1}^J \right) = \left( \{ R^{bs}_M, k^{bs}_M \}_{j=1}^J \right).
\]  

(19)

The proof is standard and omitted. Intuitively, the monopoly bank utilizes all the deposits; allocates funds symmetrically among firms (17) to maximize the aggregate production, given the concavity of the production function; and obtains all the value added, which is the socially optimal return $A$ after internalizing externality (18).

**B. Competitive Second Stage with Strategic Price Adjustment**

Without some institutional setups, no Nash equilibrium exists in the second stage with more than one bank. This is because, when the funding and lending is tightly connected, banks would face the same situation as firms would in the economy without intermediaries, the case in which no equilibrium exists as shown later in Section IV. Essentially, the ability to contract on deposit and loan amounts enables a bank to take any strategy that a firm would take in the economy without intermediaries. This differs from Stahl II (1988) and Yanelle (1998) in which intermediaries compete only in price. However, the expanded contract space in this paper is in line with a broader general equilibrium analysis with private information, such as Prescott and Townsend (1984 a, b).

Intuitively, the inherent lack of an equilibrium comes from the dilemma that banks face: Two opposite strategies look profitable. First, a bank wants to compete aggressively in the deposit market to become a monopolist. Second, a bank wants to limit deposit intake and invest a small amount in a firm to free ride on other firms’ investments. In both strategies, banks’ profits rely on other banks’ actions. On the one hand, if other banks offer deposit rates less than the monopoly loan rate, the first strategy brings a higher profit. On the other hand, if other banks offer deposit rates as high as the monopoly loan rate, the second strategy brings a higher profit. In sum, until the deposit rates are bid up to the monopoly loan rate, banks want to compete aggressively, but at that rate, banks want to shrink their size relative to others.

If an opportunity is given, agents in this economy must be willing to come up with a realistic institutional setup that can support a Nash equilibrium, in order to reduce the severe uncertainty. But, what kind of institutional setup can assure existence of a Nash equilibrium? Either of the two opposite strategies needs to be eliminated for banks to be able to make a clear decision on their deposit market strategies. It appears difficult to discourage banks from taking the first strategy (seeking more deposits), as the monopoly loan rate $A$ is always higher than other feasible rates. In contrast, banks may be easily discouraged from taking the second strategy (limiting deposit intake) if banks can adjust their fund positions using an interbank market. If so, the unique equilibrium strategy would be determined at least in the deposit
market, and the deposit market outcome could dictate the overall equilibrium candidate.

There can be many mechanisms that support a Nash equilibrium. Indeed, there is a general condition that even ensures a unique equilibrium outcome, which I discuss in Section III.F. Before doing so, however, I start with a specific mechanism as an example.

In the mechanism I propose, banks are allowed to adjust funds among themselves using an interbank market. This makes it possible for a bank to lend more than or less than deposits it collected. To clear the interbank market, banks are allowed to have free-recontracting opportunities to adjust price. This mechanism follows the spirit of the Walrasian tatonnement process in which economic agents try to find the right price to clear the market. Because the tatonnement process for a Walrasian equilibrium is not based on strategic behaviors, I propose a similar, but new, process that is consistent with strategic moves, and name it strategic tatonnement. In its simplest form, two sessions each day, morning and afternoon, are open for the loan and interbank markets. Banks can use the afternoon session as a punishment phase to support any target returns between the technologically feasible highest return and the Walrasian interest rate.

Formally, strategic tatonnement is defined as a series of substages, or sessions, that are repeated many times, possibly infinitely. Strategic tatonnement starts after the deposit market, which determines deposit rate \( r_{hi} \) and amount \( s_{hi} \). The deposit market outcome is denoted by \( z_D = ( \{ r_{hi}, s_{hi} \}_{(h,i)=(1,1)} ) \). The set of active banks depends on the deposit market outcome and is expressed as \( D(z_D) \equiv \{ h \in H : \sum_{i=1}^{I} s_{hi} > 0 \} \). The number of active banks is denoted by \#D. If \( D(z_D) \) is a singleton (i.e., \#D = 1), the loan market is monopolized, and otherwise (i.e., \#D ≥ 2), it is strategically competitive.

In each session \( \tau \in \{ 1, 2, \cdots, T \} \), there are five phases \( (p = 1, 2, 3, 4, 5) \) as listed below:

1. each bank \( h \in D(z_D) \) posts a tentative loan contract \( (R_{\tau h j}^b, k_{\tau h j}^b) \) to each firm \( j \in J \);

2. firms submit their tentative decisions \( (R_{\tau h j}^f, k_{\tau h j}^f) \) on offered contracts to banks;

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17This assumption is obviously consistent with banks’ behavior in the real world. Banks do not limit deposits based on how much they can lend, but rather adjust their fund size using the interbank market, so that deposit amounts do not strictly restrict lending operations.

18See, for example, Negishi (1987) as a reference.

19This procedure somewhat resembles the Groves-Clarke mechanism to finance public goods, but the problem here is not public goods provided by a government but rather private goods with externality provided by many private agents. Moreover, strategic tatonnement is a decentralized implementation to internalize externality, not a centralized one. In this sense, this paper is related to Bisin and Gottardi’s (2006) work on general equilibrium analysis with adverse selection in which consumption externality arises. However, their approach is different, as they expand the commodity space to include the rights to consume in order to internalize the associated negative consumption externality.

20Since the unique equilibrium outcome is given by pure strategies (Theorem 2 below), I focus on pure strategies.
3. banks submit tentative interbank rates and net borrowing amounts \((\rho^{b}_{\tau h}, B^{b}_{\tau h})\) to the interbank market, and a tentative match of demand and supply in the interbank market is undertaken;

4. if a bank satisfies its profit based on tentative matches in the loan and the interbank market, it sends a confirmation letter to each firm to accept the firm’s response, denoted as \(d^{b}_{\tau h j} = 1\) (otherwise no confirmation letter \(d^{b}_{\tau h j} = 0\)); and

5. a firm responds to the confirmation letter from a bank, denoted as \(d^{f}_{\tau h j} = 0\) if it rejects the letter and \(d^{f}_{\tau h j} = 1\) if it accepts, but a firm has no choice other than accepting the bank’s decision \(d^{f}_{\tau h j} = 1\) in case of no letter sent from bank \(h\).

Status of agreements between bank \(h\) and firm \(j\) is an outcome of session \(\tau\) and is denoted by \(d_{\tau h j} \in \{0, 1\}\): \(d_{\tau h j} = 1\) if \(d^{b}_{\tau h j} = d^{f}_{\tau h j} = 1\) and otherwise \(d_{\tau h j} = 0\). If all banks and firms agree, the strategic tâtonnement ends; otherwise, the next session begins. Once a bank and firm accept a contract, they must honor it. In later sessions, no matter what alternatives exist, both parties are assumed to submit the agreed contract to each other and repeat confirmation and acceptance.\(^{21}\)

To allow banks to adjust price and quantity, I assume that at least two sessions, \(T \geq 2\), exist. Here, \(T = 2\) suggests that morning and afternoon sessions exist in the interbank market. Other finite \(T\) suggests a longer, but similar, situation. Infinite \(T\) implies that banks and firms talk continuously all day. Even if \(T\) is infinite, it still is contained in one period. After the strategic tâtonnement ends, transactions are made based on the agreed contracts, and profits of banks and firms are realized. If a bank-firm pair does not reach an agreement, then there is no transaction of capital within the pair.

The interbank market is assumed to be a centralized, multilateral clearing system among banks without an auctioneer or a central bank. Each bank picks a specific interest rate \(\rho^{b}_{\tau h} \in \mathbb{R}_+\) and indicates its willingness to borrow \(B^{b}_{\tau h} \in \mathbb{R}\). Here, negative value means supply of funds.\(^{22}\) The interbank market must clear in an equilibrium. I define the set of active banks that submit the interbank market rate \(\rho\) as \(\hat{D}_{\tau}(\rho) \equiv \{h \in D(Z_D) | \rho^{b}_{\tau h} = \rho\}\). Using this notation, aggregate net borrowing at each interbank market rate \(\rho \in \mathbb{R}_+\) can be

\(^{21}\)This assumption is not restrictive as an abstract description of negotiation over loan terms. It is made only because of notational simplicity, compared with an alternative assumption that a specific bank-firm pair withdraws the process when they agree. In either specification, banks and firms face essentially the same decision. Note that banks and firms have a choice not to agree to a specific loan contract if they think there would be a better opportunity in later sessions.

\(^{22}\)Instead of picking one specific interest rate, it could be formulated that each bank submit a borrowing function for all the possible interest rates in the real value. It could also be formulated with a bilateral clearing system in which contracts can be indexed by lenders’ and borrowers’ identities denoted by subscripts. These changes would make the model more complex but would not affect the main results.
defined as
\[
\overline{B}_\tau(\rho) \equiv \sum_{h \in \tilde{D}(\rho)} B^b_{\tau h}.
\] (20)

With the interbank borrowing, each bank needs to satisfy the resource constraint (i.e., the bank balance sheet condition):
\[
\sum_{j=1}^J k^f_{\tau hj} \leq \sum_{i=1}^I s_{hi} + B^b_{\tau h}.
\] (21)

The interbank-market-clearing condition is now written as follows. For all \(\rho \in \mathbb{R}_+\), there exists \(\tau^* \leq T\) such that
\[
\overline{B}_{\tau^*}(\rho) = 0.
\] (22)

Because the agreement is defined, such that all agreed bank-firm pairs repeat their agreed contracts, the interbank market always clears in any session once it clears in session \(\tau^*\); i.e., for any \(\hat{\tau} \geq \tau^*\), \(\overline{B}_{\hat{\tau}}(\rho) = 0\) for all \(\rho \in \mathbb{R}_+\).

In summary, in each session \(\tau\), the strategy of bank \(h\) is defined as
\[
z^b_{\tau h} \equiv \{\tilde{z}^b_{\tau h}, d^b_{\tau h}, (\rho^b_{\tau h}, B^b_{\tau h})\},
\] (23)

where \(\tilde{z}^b_{\tau h}\) is a bank’s strategy in the loan market (\(\{R^b_{\tau hj}, k^b_{\tau hj}\}_{j=1}^J\)), and \(d^b_{\tau h}\) is a set of bank \(h\)’s confirmation strategies toward all firms (\(\{d^b_{\tau hj}\}_{j=1}^J\)). Thus, the strategy set is given by
\[
Z^b \equiv (\mathbb{R}_+ \cup \{N.S.\})^2J \times \{0, 1\}^J \times (\mathbb{R}_+ \cup \{N.S.\})^2.
\] (24)

Note that this strategy set is the same for every bank \(h\) and session \(\tau\). I also use the vector notation \(z^b_{\tau} \equiv (\{z^b_{\tau h}\}_{h=1}^H)\) for the strategies of all banks in session \(\tau\) and \(\tilde{z}^b_{\tau} \equiv (\{\tilde{z}^b_{\tau h}\}_{h=1}^H)\) for the strategies on loan contracts.\(^{23}\)

Firm \(j\)’s strategy, when it faces offers from banks in session \(\tau\), is to choose
\[
z^f_{\tau j} \equiv (\{R^f_{\tau hj}, k^f_{\tau hj}, d^f_{\tau hj}\}_{h \in H})\) from its strategy set \(Z^f \equiv \mathbb{R}_+^2J \times \{0, 1\}^H\). Similar to the notation for banks, I use the vector notation \(z^f_{\tau} \equiv (\{z^f_{\tau j}\}_{j=1}^J)\) for the strategies of all firms in session \(\tau\).

The strategy set for a firm is constrained by banks’ strategies \(z^b_{\tau}\). The constrained choice set is written as \(G^f_{j}(z^b_{\tau})\). I assume that “reject,” \(k^f_{\tau hj} = 0\), is always in the choice set. Let \(G^f_{hj}(z^b_{\tau hj})\) be an element of \(G^f_{j}(z^b_{\tau})\) corresponding to the constrained choice set of firm \(j\) with

\(^{23}\)For simplicity, I hereafter focus on the case in which all the banks are active, \(\#D = H\). I will make clear those situations when the case \(\#D < H\) should be treated carefully.
respect to bank $h$:

$$G_{hj}^f(z_{rhj}) \equiv \tilde{G}_{hj}^f(z_{rhj}) \times \{0, 1\}, \quad (25)$$

where $\tilde{G}_{hj}^f(z_{rhj})$ is the constrained choice set in the loan market for the loan contract $z_{rhj}^f \equiv (R_{rhj}^f, k_{rhj}^f)$. This constrained choice set for a firm facing bank $h$’s offer is defined as the same as in (12), in the case for a firm facing a monopoly bank’s offer. The constrained choice set of each firm facing many banks’ offers is defined as the Cartesian product of $G_{hj}^f$ over $h \in H$:

$$G_j^f(z_{rh}^b) \equiv G_{1j}^f(z_{r1j}^b) \times G_{2j}^f(z_{r2j}^b) \times \cdots \times G_{Hj}^f(z_{rHj}^b). \quad (26)$$

Before formally defining a session, a strategy history needs to be defined because strategies in session $\tau$ can be conditional on their history. Note that $(z_{r}^b, z_{r}^f)$ represents all the strategies of banks and firms in session $\tau$.

**Definition 2.** A history $(z_{r}^b, z_{r}^f)^{\tau-1}$ for session $\tau = 2, \cdots, \infty$ denotes strategy sequences before session $\tau$ for banks and firms; that is,

$$(z_{r}^b, z_{r}^f)^{\tau-1} \equiv (z_{r1}^b, z_{r1}^f), (z_{r2}^b, z_{r2}^f), \cdots, (z_{r\tau-1}^b, z_{r\tau-1}^f).$$

The corresponding space of the history is $(Z_{r}^b \times Z_{r}^f)^{\tau-1}$ for $\tau = 2, \cdots, \infty$. The first session has the empty history.

Recall that $p = 1, 2, 3, 4, 5$ denotes five phases within a session. Let $P(p)$ denote a player function that assigns players in phase $p$ and $G(p)$ denote constrained strategy space for players $P(p)$. Now, each session $\tau$ can be defined formally as

$$\Phi_{\tau}(z_D, (z_{r}^b, z_{r}^f)^{\tau-1}) \equiv ((H, J), p, P, G, d_{rhj}), \quad (27)$$

given deposit market outcome $z_D$ and loan- and interbank-market history $(z_{r}^b, z_{r}^f)^{\tau-1}$.\(^{24}\)

A set of strategy $(\{z_{r}^b, z_{r}^f\}_{\tau=1}^T)$ brings an outcome. Outcome values are written without any superscripts as before and, here, also without session subscript $\tau$. If the loan contract between bank $h$ and $j$ is agreed on at session $\overline{\tau}$ (i.e., $d_{\overline{\tau}hj} = 1$), then the outcome is

$$(R_{\overline{\tau}hj}, k_{\overline{\tau}hj}) = (R_{\overline{\tau}hj}^b, k_{\overline{\tau}hj}^b) = (R_{\overline{\tau}hj}^f, k_{\overline{\tau}hj}^f). \quad (28)$$

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\(^{24}\)This formulation is similar to an extensive game with perfect information and simultaneous moves, though it is not a game because the payoff functions are not defined until all sessions are completed. If simultaneous opening of both the deposit and loan markets were assumed, a loan market strategy must be decided before knowing the size of deposits collected by each bank. Instead of formulating the subgame conditional on the realized deposit market outcome, the loan market could be formulated conditional on expectations for all possible equilibrium deposit market strategies. This approach can also be applied to the two-stage game in which the loan market is open in the first stage, but banks and firms cannot commit to honor contracts when banks fail to raise sufficient funds. Note that Stahl II’s (1988) analysis of his loan-market-first game is different, as it assumes that a firm-bank pair must honor the loan contract in any circumstances without knowing the banks’ fund positions. These cases can be analyzed similarly; see further discussions in Section III.E.
Similarly, if all banks and firms agree at session $\tilde{\tau}$, then the interbank market contracts at session $\tilde{\tau}$ represent an outcome; i.e., if $d_{\tau_{hj}} = 1$ for all $h \in D(z_D)$ and $j \in (1, 2, \cdots, J)$,

$$(\rho_h, B_h) = (\rho_{\tau_{hj}}^b, B_{\tau_{hj}}^b).$$

The outcome of the competitive second stage is expressed as $z_L \equiv \{\{ R_{hj}, k_{hj} \}_{(h,j)=(1,1)}, \{ \rho_h, B_h \}_{h=1}^H \}$. This is a function of a set of strategy $\{ \{ z_{\tau_{hj}}^b, z_{\tau_{hj}}^f \} \}_{\tau=1}^T$. Given the deposit market outcome $z_D$, a bank maximizes its profit by choosing its loan market strategy $z_{\tau_{hj}}^b$ for all $\tau$,

$$
\pi_{Lh}^b = \pi_{Lh}^b(\{ z_{\tau_{hj}}^b \}_{\tau=1}^T \{ z_{\tau_{hj}}^b, z_{\tau_{hj}}^f \}_{\tau=1}^T, z_D) = \sum_{j=1}^J R_{hj} k_{hj} - \rho_h B_h - \sum_{i=1}^I r_{hi} s_{hi}.
$$

Also, a firm chooses its loan market strategy $z_{\tau_{hj}}^f$, for all $\tau$, to maximize its profit $\pi_{fj}^f$, defined similarly based on outcomes, as in (3).

**Definition 3.** Strategic tâtonnement is the competitive second-stage game and is a settlement procedure represented as $T$-times (possibly infinitely) repeated sessions. More specifically, it consists of $H$ banks and $J$ firms, the set of all possible histories, a collection of sessions as a function of history, and final profits for banks and firms, given the deposit market outcome $z_D$.

$$
\Gamma^T_C(z_D) \equiv \{(H, J), (Z^b \times Z^f)^T, \{\Phi^\tau_c(z_D, (z^b, z^f)^{\tau-1})\}_{\tau=1}^T(\pi_L^b, \pi_f^f)\}.
$$

**Definition 4.** An equilibrium of the competitive second stage is a set of strategies $\{ z_{\tau_{hj}}^b, z_{\tau_{hj}}^f \}_{\tau=1}^T$ that is a Nash equilibrium of the game $\Gamma^T_C$ and clears the interbank market; i.e., there exists $\tau^* \leq T$ that satisfies (22).

Based on equilibrium strategies, an equilibrium outcome is denoted by $z_L^* \equiv \{ R_{hj}^*, k_{hj}^* \}_{(h,j)=(1,1)}, \{ \rho_h^*, B_h^* \}_{h=1}^H$ with an agreed session $\tau^*$.

**C. Loan Market Equilibrium with Strategic Tâtonnement**

I guess that banks’ strategies over the strategic tâtonnement should consist of a target contract, a detection mechanism, and a punishment contract. I do not intend to describe all possible equilibrium strategies, because the objective here is to show an example that ensures

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25To be consistent with the rest of the paper, it might be better to formulate the strategic tâtonnement process in a strategic form in which, at the outset, firms and banks pick their strategies for all sessions, $\tau = 1$ to $T$, for all possible realizations of histories of strategies. Here, it is defined in an extensive form, because the description of the game is simpler and intuitive. Besides, any extensive form game can be converted into a strategic form.
the existence of an equilibrium. To identify a set of equilibrium strategies, I use a “guess and verify” method.

Intuition behind the guess is as follows. Because banks have incentives to exploit as much revenue from firms as possible, they prefer to offer a take-it-or-leave-it contract that specifies both loan rate and amount. Some banks would take into account the spillover effect among their client firms and force them to invest more than suggested by the private marginal product of capital. However, other banks would take advantage of the externality and make profits by offering a small amount of investment with a slightly higher loan rate to a firm. This free-riding strategy dramatically lowers the return from the large-investment strategy. Apparently, banks want to detect the deviation. The interbank market can be used as a detection mechanism: The interbank market will not clear with any free-riding deviations, because deviants always use less capital than others. Banks punish such deviations by changing their offers after they observe that the interbank market does not clear.

I define two contracts in the loan market, one as a candidate for a punishment strategy and the other as a candidate for a target strategy, and call them the Walrasian contract and the Pareto-optimal contract, respectively. The Walrasian contract in the loan market is denoted by:

\[ z_w \equiv (\alpha A, N. S.). \]  

(32)

The loan rate is the same as the Walrasian equilibrium rate, and the amount is not specified so that the loan market always clears at this rate. The Pareto-optimal contract is defined similarly as

\[ z_p \equiv \left( A, \frac{S}{J} \right), \]  

(33)

where \( S \) denotes the aggregate savings already collected in the deposit market. The loan rate is the same as the social planner’s return from the aggregate production function, and the loan amount is equal to the deposits per firm.

With the Walrasian contract, banks and firms can achieve the same loan market outcome as in the Walrasian equilibrium in which the private marginal product of capital is equal to the loan rate. Therefore, it is neither surprising nor difficult to show that the Walrasian contract is an equilibrium contract, and the proof for Lemma 2, below, is omitted.

**Lemma 2.** Repeatedly offering the Walrasian contract in consecutive sessions from the first session, or after a disagreed session, constitutes a Nash equilibrium in the competitive second stage.

Although the Walrasian contract may prevail in an equilibrium in the loan market, as shown in Lemma 2 above, a bank can use a different strategy in the strategic tâtonnement to
achieve a higher return. With a target loan rate $\phi A$, I define a general target loan contract as

$$z_\phi \equiv \left( \phi A, \frac{S}{J} \right).$$ \hspace{1cm} (34)

For now, the target loan rate is assumed to be somewhere between the Walrasian and the Pareto-optimal rate, $\phi \in [\alpha, 1]$. The target loan amount is the same as in the Pareto-optimal allocation.

**Lemma 3.** The following strategies constitute a Nash equilibrium for $T < \infty$ and a subgame perfect equilibrium for $T = \infty$ in the competitive second stage:

(i) every bank offers the same target contract unless some banks deviate in the previous session; i.e.,

$$z_{b,\tau h_j}^b = z_\phi \quad \text{if } \tau = 1 \text{ or if } \tau > 1 \text{ with } z_{b,\tau - 1 l_j}^b = z_\phi \text{ for all } l \in -h,$$

$$= z_w \quad \text{otherwise};$$ \hspace{1cm} (35)

(ii) when all banks adopt the loan market strategy (35), only two cases happen: either (a) a firm receives only $z_\phi$ offers from all banks, and in this case, the firm’s best response is

$$z_{f,\tau h_j}^f = \left( \phi A, \frac{S}{J} \right), \quad \text{accepting the offer from one bank (e.g., bank } h), \text{ and}$$

$$z_{f,\tau l_j}^f = (\phi A, 0), \quad \text{rejecting the offers from other banks } l \neq h;$$ \hspace{1cm} (36)

or (b) a firm receives at least one offer of the Walrasian contract $z_w$ (e.g., from bank $h$), and in this case, the firm’s best response is

$$z_{f,\tau h_j}^f = \left( \alpha A, \frac{S}{J} \right), \quad \text{accepting the Walrasian contract offered from bank } h, \text{ and}$$

$$z_{f,\tau l_j}^f = (R_{l j}^b, 0), \quad \text{rejecting the offers from other banks } l \neq h;$$ \hspace{1cm} (37)

(iii) when banks and firms adopt loan-market strategies described above (35)–(37), an equilibrium strategy in the interbank market for bank $h$ is

$$(\rho_{b,\tau h}, B_{b,\tau h}^b) = \left( \phi A, \sum_{j=1}^J k_{f,\tau h_j}^f - s_h \right) \quad \text{when } z_{b,\tau h_j}^b = z_\phi,$$

$$= \left( \alpha A, \sum_{j=1}^J k_{f,\tau h_j}^f - s_h \right) \quad \text{otherwise (when } z_{b,\tau h_j}^b = z_w);$$ \hspace{1cm} (38)

(iv) when banks and firms adopt the loan-market and interbank-market strategies described above (35)–(38), an equilibrium strategy for a bank in the banks’ confirmation phase is to
confirm only when the interbank market clears; i.e.,

\[ d^b_{rhj} = \begin{cases} 1 & \text{if } B_r(\rho) = 0 \text{ for all } \rho \in \mathbb{R}_+, \\ 0 & \text{otherwise} \end{cases} \]  

(39)

(v) finally, when banks and firms adopt the strategies described above (35)–(39) in the loan market, the interbank market, and the banks’ confirmation phase, an equilibrium strategy for a firm in the firms’ confirmation phase is to confirm the acceptance letter always and, in case of no letter, to agree with no transaction automatically; i.e.,

\[ d^f_{rhj} = 1. \]  

(40)

The formal proof is provided in online Appendix II. The main issue here is to find conditions for the existence of a Nash equilibrium, not to use refinement techniques to pick one among many Nash equilibria. This is consistent with many other papers dealing with a discontinuous game (see online Appendix III). However, the economy may still face instability, even with the strategic tâtonnement, in the sense that the Nash equilibrium described above is not robust to subgame perfection for finite T.\(^{26}\)

The target loan rate range \([\alpha A, A]\) has been assumed but, indeed, this must be true with the equilibrium strategies described in Lemma 3. First, \(\phi A > A\) is not feasible as a target contract, because \(A\) is the highest loan rate technologically possible when firms invest the same amount of capital as specified in the target contract. Second, \(\phi A < \alpha A\) does not work either, as the punishment strategy is \(A\).

The set of equilibrium strategies in Lemma 3 produces immediate clearing of the loan and interbank markets. The equilibrium outcome is summarized as follows:

(i) no arbitrage of the loan and interbank rates: for all \(h\) and \(j\), \(R_{hj} = \rho_h = \phi A\);

(ii) upper and lower bounds of the equilibrium loan rate: \(\alpha A \leq \phi A \leq A\);

(iii) symmetrical capital allocation for each firm, but not necessarily among banks:

\[ k_{hj} = \frac{S}{J}, \text{ for some } h, \text{ and } k_{lj} = 0, \text{ for } l \neq h; \text{ and} \]

(iv) the interbank market clears: for all \(\rho \in \mathbb{R}_+, B(\rho) = 0.\)\(^{27}\)

Because the target loan rate \(\phi A\) can be any number between \(\alpha A\) and \(A\), Lemma 3 implies that, when two or more banks are active, many Nash equilibrium outcomes exist in the

\(^{26}\)The only equilibrium in the last session \(T < \infty\) is the one with the Walrasian contract. See more detailed analysis in the proof of Lemma 3 (in online Appendix II). Note that I invented strategic tâtonnement as a Nash equilibrium mechanism; there is a possibility that some institutional setups implement allocations based on a subgame perfect equilibrium, even for \(T < \infty\).

\(^{27}\)Here, bank \(h\)’s net borrowing from the interbank market is given by \(B_h = \sum_{j=1}^{J} k_{hj} - s_h\) at the interbank rate \(\rho = \phi A\). For other interest rates, there is neither demand nor supply of funds in the interbank market.
second stage for $T < \infty$, and many subgame perfect equilibrium outcomes exist for $T = \infty$. Moreover, there can be many strategies, other than those described in Lemma 3, to support the same equilibrium outcome in the loan market. Also, any mixed combination of these strategies can constitute an equilibrium; for example, the punishment loan rate may be different from $\alpha A$, or a more complex scheme can work. Again, however, the objective is not to list all the possible equilibria in the competitive second stage but to show the existence of one Nash equilibrium. Nonetheless, more general results will be shown in Section III.F.

**D. Deposit Market**

In the first stage, the deposit market, the strategy of bank $h$ to consumer $i$ is to post a deposit contract that specifies deposit rate and amount, $\z^b_{Dhi} \equiv (r^b_{hi}, s^b_{hi})$. The strategy set is defined as

$$Z^b_D \equiv (\mathbb{R}_+ \cup \{N.S.\})^2.$$  

A consumer $i$’s strategy is $\z^c_i = (\{r^c_{hi}, s^c_{hi}\})_{h=1}^H$, chosen from the strategy set $Z^c \equiv \mathbb{R}_+^{2H}$. However, this strategy set is constrained by $\z^b_{Di}$, i.e., all banks’ strategies to consumer $i$. The constrained choice set of consumer $i$ is expressed as $G^c_{hi}(\z^b_{Dhi})$. Let $G^c_{hi}(\z^b_{Dhi})$ be an element of $G^c_{hi}(\z^b_{Dhi})$ corresponding to the constrained choice set of consumer $i$ facing bank $h$’s offer. I assume that zero deposit in bank $h$, $s^b_{hi} = 0$, is always in the choice set:

$$G^c_{hi}(\z^b_{Dhi}) \equiv r^b_{hi} \times \mathbb{R}_+ \quad \text{if bank } h \text{ specifies } r^b_{hi} \text{ only,}$$

$$\equiv \mathbb{R}_+ \times (s^b_{hi} \cup \{0\}) \quad \text{if bank } h \text{ specifies } s^b_{hi} \text{ only, and}$$

$$\equiv r^b_{hi} \times (s^b_{hi} \cup \{0\}) \quad \text{if bank } h \text{ specifies both } r^b_{hi} \text{ and } s^b_{hi}. \quad (42)$$

Note that the choice set of the last case is either $(r^b_{hi}, s^b_{hi})$ or $(r^b_{hi}, 0)$; i.e., a consumer replies either to “accept” or “reject” the offer. The set constrained by offers from all banks is defined as the Cartesian product of $G^c_{hi}$ over $h \in H$:

$$G^c_i(\z^b_{Di}) \equiv G^c_{i1}(\z^b_{D1i}) \times G^c_{i2}(\z^b_{D2i}) \times \cdots \times G^c_{Hi}(\z^b_{DHi}). \quad (43)$$

The feasible set from which a consumer chooses her savings today is a combination of the

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28 Equity-type contracts, whose returns depend on outcome, are not worth considering, because they will be driven away by debt contracts that promise to repay the expected return of equity-type contracts. Since households are risk averse and banks are risk neutral, households prefer debt contracts to equity-type contracts with the same expected return, and banks are indifferent between them.
constrained choice set and the budget set,
\[ \Delta_{Bi} = \Delta_B(m_i, z_{Di}^b) \equiv \left\{ z_i^c : z_i^c \in G_i^c(z_{Di}^b) \quad \text{and} \quad \sum_{j=1}^J s_i^c \in [0, m_i] \right\}. \] (44)

An outcome of the first stage for \( i \)-th consumer and \( h \)-th bank pair is denoted without superscript by \( z_{Dhi} = (r_{hi}, s_{hi}) \); the outcome for consumer \( i \) by \( z_{Di} = (r_i, s_i) \) (i.e., \( r_i \) is the weighted average deposit rate and \( s_i \) is the total savings); and the outcome vector for all consumers by \( z_D = (z_{D1}, z_{D2}, \ldots, z_{DI}) \), as a result of all consumers’ strategies \( z^c \equiv \{z_i^c\}_{i=1}^I \) and all banks’ strategies \( z^b_D \equiv \{z_{Di}^b\}_{i=1}^I \).

Let \( z_2 \) denote an outcome of the second stage. It is either monopolistic outcome \( z_M \) or strategically competitive outcome \( z_L \). For a given second-stage outcome, bank \( h \)’s objective in the deposit market is to maximize its profit by choosing its strategy \( z_{Dh}^b \),
\[ \pi_h^b = \pi_h^b(z_{Dh}^b|z_{D-h}^b, z^c, z_2) = \sum_{j=1}^J R_{hj} k_{hj} - \rho_h B_h - \sum_{i=1}^I r_{hi} s_{hi}. \] (45)

Both firm and bank profits are affected by aggregate savings. The profit income of a consumer \( w_i \), defined in (7), becomes a function of all consumers’ strategy, including her own, in addition to all banks’ and firms’ strategies. For the sake of simplicity, I assume that the number of households are very large and the ownership is dispersed. This means that when a consumer decides her savings, she only cares about direct returns from savings without considering its effect on her profit income. In other words, I write the profit income as a function, \( w_i(z_{-i}^c, z_D^b, z_2) \).

Based on definitions (4) to (8), the utility of consumer \( i \) can be expressed in terms of her own savings \( s_i^c \) and savings return \( r_i^c \), given profit income \( w_i \) and initial wealth \( m_i \):
\[ V(z_i^c|z_{-i}^c, z_D^b, z_2, m_i) = u(m_i - s_i^c) + \beta u(r_i^c s_i^c + w_i(z_{-i}^c, z_D^b, z_2)). \] (46)

The deposit market is defined formally as the first stage of the game as follows.

**Definition 5.** Given the initial wealth distribution \( m \) and the second-stage outcome \( z_2 \), let \( \Gamma_1(m, z_2) \) denote the deposit market, the first stage of the whole game. It consists of \( H \) banks and \( I \) consumers, their constrained strategy sets, the lifetime utility of consumers, and the profit of banks:
\[ \Gamma_1(m, z_2) \equiv ((H, I), (z_D^b, \Delta_{Bi}), (\pi^b, V)|m, z_2). \] (47)

The savings function in the Walrasian economy is another key concept to understand the analysis below. I use \( s(m_i, (r, N.S.)) \) to represent the personal savings function when a
consumer, with wealth $m_i$, faces the only one financial contract, which does not specify savings amount but specifies the savings return $r$. This is equal to the personal savings function in the Walrasian economy in which auctioneer offers $r$.\textsuperscript{29} When this contract $(r, N.S.)$ is the only one offered by all the banks, the aggregate Walrasian savings function $S$ is defined as the sum of the personal savings for consumers with wealth distribution $m$, 

$$S(m, (r, N.S.)) \equiv \sum_{i=1}^I s(m_i, (r, N.S.)).$$

E. Equilibrium of the Whole Game

Definition 6. A second stage is the following game, given a deposit market outcome $z_D$:

$$\Gamma^T_2(z_D) \equiv \Gamma_M(z_D) \quad \text{if } D(z_D) \text{ is a singleton, and}$$

$$\equiv \Gamma^T_C(z_D) \quad \text{otherwise.}$$

Definition 7. A strategically intermediated economy is the game $\Gamma^T$, given wealth distribution $m$. It consists of the following elements:

- the first stage, $\Gamma_1(m, z_2)$;
- the set of all possible histories for the second stage, which is all possible strategies in the first stage, $(Z^b_D)^H \times (Z^c_I)$; and
- the second stage, $\Gamma^T_2(z_D)$.

In sum,

$$\Gamma^T(m) \equiv (\Gamma_1(m, z_2), (Z^b_D)^H \times (Z^c_I), \Gamma^T_2(z_D)).$$

Definition 8. An equilibrium of the strategically intermediated economy is a Nash equilibrium of the game $\Gamma^T(m)$. It must satisfy two conditions:

(i) given the equilibrium second-stage outcome $(z^*_M, z^*_L)$, deposit market strategies, $(z^{ca}_c, z^{bs}_D)$,

\textsuperscript{29}See Appendix I for the optimal savings decision by a consumer in the Walrasian equilibrium.\n
\textsuperscript{30}If the number of consumers were not large, the personal savings would depend on others’ savings and own savings via profit income from banks (proportional to ownership) in the economy. Then, it should be defined as $s(m_i, (r, N.S.), s_{-i}, \{\psi^{b}_{hi}\}_{h=1}^H)$. The aggregate savings function should be formulated in the same way as $S(m_i, (r, N.S.), \{\psi^{b}_{hi}\}_{(h,i)=(1,1)} = (1,1)) \equiv \sum_{i=1}^I s(m_i, (r, N.S.), s_{-i}, \{\psi^{b}_{hi}\}_{h=1}^H)$. Both the personal and aggregate savings functions might take slightly different values from those in the Walrasian economy (with one Walrasian auctioneer). However, these values in this economy would coincide with those in the Walrasian economy if each person does not take into account the effect of own savings on the profit income. This happens when the number of consumers are infinitely many. Note that firm profit income does not enter into the savings function because consumers cannot directly affect the firms’ profit income thanks to bank intermediation. To compare key results associated with several regimes considered in this paper, I assume that a large number of consumers exist with dispersed ownership and, accordingly, that a consumer chooses her savings without considering its effects on her profit income in any regimes.
are Nash equilibrium strategies of the first stage and \( z^*_D \) is the outcome; and
(ii) given the equilibrium first-stage outcome \( z^*_D \), loan market strategies, \( \{ z^{b*}_T, z^{f*}_T \}_{T=1} \), are Nash equilibrium strategies of the competitive second stage with associated outcome \( z^*_L \), and
\( \{ z^{M*}_{Mj}, z^{F*}_{Mj} \}_{j=1} \) are Nash equilibrium strategies of the second stage with a monopoly bank with associated outcome \( z^*_M \).

**Theorem 1. [Existence of an Equilibrium]** An equilibrium exists in the bank-intermediated economy \( \Gamma^T(m) \) with strategic tâtonnement. An equilibrium outcome is Pareto optimal with many active banks. It is based on the set of strategies described in Lemma 3 and characterized as follows:

(i) banks’ equilibrium offers specify the deposit rate \( r^{b}_{hi} \) equal to \( A \);
(ii) not specifying deposit amounts (i.e., \( s^{b}_{hi} = \{N.S.\} \)) is a weakly dominant strategy;\(^{31}\)
(iii) each depositor faces the deposit contract offer \((A, N.S.)\) and makes deposits \( s(m_i, (A, N.S.)) \), which sums up to the aggregate savings \( S(m, (A, N.S.)) \);
(iv) the equilibrium loan offer by any bank in all the sessions of the competitive second stage is \((A, S(m, (A, N.S.))/J)\), the same as the equilibrium contract in the second stage with a monopolist bank; and
(v) firms accept this contract in the first session (and thereafter).

**Proof.** Lemma 3 identifies that the Pareto optimal contract \((A, S(m, (A, N.S.))/J)\) is a loan-market equilibrium strategy in all the sessions of the competitive second stage. This is not conditional on the deposits taken by each bank in the first stage. Thus, at the deposit rate \( A \), banks weakly prefer not restricting deposit amount. Since the equilibrium loan rate is \( A \), banks have no incentive to solicit more deposits with a deposit rate higher than \( A \). A deposit rate less than \( A \) cannot be an equilibrium either; otherwise, there would exist an arbitrage opportunity.

Because \( \phi \) can be any number between \( \alpha \) and 1 in Lemma 3, there are multiple equilibrium outcomes in the loan market. However, competition in the deposit market selects unique Nash equilibrium outcome, \( \phi A = A \) case, for the whole game. Still, within the proposed set of strategies in Lemma 3, multiple Nash equilibria may exist only because the deposit and loan market shares of each bank are not uniquely determined. But, equilibrium outcome is unique in terms of economic consequences; i.e., the deposit rate for and amounts from each household and the loan rate and amounts for each firm as well as (zero) profit for each bank.

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\(^{31}\)This property implies that competition in the deposit market is likely to become à la Bertrand, competition in price only. The strategy that specifies a depositor’s willingness to supply at rate \( A \) (i.e., \( s^{b}_{hi} = s(m_i, (A, N.S.)) \)) can be regarded as essentially the same strategy that does not specify deposit amount under deposit rate at \( A \) (i.e., \((A, N.S.)\)). For more general Nash equilibrium strategies, see the proof of Theorem 2 (in online Appendix II).
There may exist equilibrium strategies other than those described in Lemma 3. However, Theorem 2 in the next section assures the uniqueness of the equilibrium outcome in the economy with *strategic tâtonnement*.

Simultaneous opening of deposit and loan markets can be analyzed in the same way, if, similar to the real-world function of the interbank market, some time is left for clearing the loan and interbank markets after the closing of deposit-taking business within a period. The equilibrium strategy is the same as before, except that the first session of the loan market would be played conditional on the expected amount of aggregate savings. Specifically, in the morning, the deposit market and the first session of *strategic tâtonnement* are open at the same time. In the first session, banks rationally expect that the aggregate savings would be at the socially optimal level and make the Pareto-optimal loan-contract offers to firms. At the end of the morning, the deposit market closes, the first session of loan offers ends, and the first session of the interbank market begins. Given the realized deposit amount and the offered loan amount, banks post any demand or supply of funds to the interbank market. If the aggregate deposit is the Pareto-optimal one, then the interbank market clears, and the proposed equilibrium allocation is achieved. Otherwise, the punishment strategy, specified in Lemma 3, would be taken in the second session of loan and interbank markets.

Further discussions of the proposed mechanism in several variants of the model are included in an earlier version of this paper. For example, a Walrasian corporate bond market would play no role if it is added to the bank intermediation in the model. Also, if the production function exhibits decreasing returns to scale (i.e., \( \eta < 1 - \alpha \) in (3)), the proposed mechanism does not support the socially optimal allocation but still supports an equilibrium that is Pareto superior to the Walrasian equilibrium.

**F. Uniqueness of the Equilibrium Outcome**

What I showed in the previous section is that, with a specific institutional setup (i.e., *strategic tâtonnement*), the set of strategies described in Lemma 3 constitute an equilibrium in the competitive second stage and then support an equilibrium for the whole game. Other equilibrium strategies may exist in the same economy with *strategic tâtonnement*. However, from the viewpoint of economic consequences, an important question is whether the equilibrium outcome described in Theorem 1 is unique, in terms of the deposit contract for each household and the loan contract for each firm. I confirm this uniqueness in this section.

A key characteristic hidden in *strategic tâtonnement* is that banks do not have to worry about consequences in the loan-market competition when they compete for deposits. This comes from two features: that banks can adjust fund size in the *interbank market* and that the interbank market is assured to clear, thanks to *free recontracting* opportunities on loan and...
interbank contracts.

To see the importance of the first feature (interbank market), assume an economy with free recontracting opportunities but without an interbank market, and consider the strategy similar to the one in Lemma 3: Unless other banks or firms deviate, a bank offers the Pareto-optimal contract; otherwise, it offers the Walrasian contract. This strategy works only when the deposit amount is assumed to be always equal for every bank offering the same deposit rate. In general, it does not work, because a bank that attracted less deposits by chance, or limited deposit intake strategically, can offer a firm a loan contract specifying a lower capital level with a slightly higher loan rate. This is possible because, without the interbank market, other banks cannot help but lend out all the deposits they took in order to maximize the profit.32

Similarly, the second feature (free recontracting) is important. In an economy with a one-shot interbank market without any recontracting opportunities, the proposed strategy does not work because a deviant bank does not have any fear of punishment.

The key characteristic (i.e., no worry for the size of collected deposits) in strategic tâtonnement can be generalized as a weak link between sources and uses of funds, which is formally defined below. I use $z^b_h$ to denote bank $h$’s strategy for a general competitive second stage with an (yet-undescribed) institutional setup; accordingly, the associated strategy space is more generally defined. Let $\lambda_h$ denote the corresponding mixed strategy in the competitive second stage, which is a probability measure. Similarly, I let $q_j$ denote firm $j$’s mixed strategy over corresponding pure strategy $z^f_j$.33 I use $z^b, z^f, \lambda, \lambda$ and $q$ to denote a vector of individual strategies (e.g., $q = (q_1, \cdots, q_J)$). Moreover, let $\lambda^{l\setminus h}$ denote a mixed strategy in which banks $h$ and $l$ exchange strategies, keeping the order of firms fixed:

$\lambda^{l\setminus h} = (\lambda_1, \cdots, \lambda_h-1, \lambda_l, \lambda_{h+1}, \cdots, \lambda_{l-1}, \lambda_h, \lambda_{l+1}, \cdots, \lambda_H);$ let $\lambda^{l\setminus h}_i$ denote its $i$-th element.

Also, let $q^{l\setminus h}$ denote a set of mixed strategies in which all firms’ strategies for bank $h$ are exchanged by their strategies for bank $l$, and let $q^{l\setminus h}_j$ denote its $j$-th element. I use superscript * to denote the equilibrium strategy as before.

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32 With the interbank market, in the punishment strategy used in Lemma 3, other banks can keep specifying the loan amount at the average capital per firm, which is not necessarily the same amount as the deposits they took. Then, other banks can attract all firms and squeeze the deviant bank to lend its fund to the interbank market at the Walrasian interest rate. See the proof for Lemma 3 (in online Appendix II).

33 A mixed strategy is chosen from a space of probability measure on the pure strategy space. More generally, let $\mathcal{B}(X)$ denote a Borel $\sigma$-algebra of pure strategy space $X$. $\Lambda(X)$ is a space of probability measure on the measurable space $(X, \mathcal{B}(X))$. I assume measurability of maximands following convention (Stinchcombe and White, 1992). Note that a mixed strategy is often defined as a probability distribution over pure strategies. A probability distribution is a probability measure on a subset of the real line (Billingsley, 1995, p. 188). As the space of the pure strategy here is multi-dimensional, including the “not specified” option, I use a more general term, probability measure, to denote a mixed strategy. This notation is in line with a tradition of general equilibrium analysis with private information (e.g., Prescott and Townsend, 1984 a, b) in which agents trade probability measures or “lotteries” on pure-strategy contracts.
Definition 9. An institutional setup creates a weak link between sources and uses of funds, if the probability of adopting a specific pure strategy as an equilibrium strategy in the competitive second stage is the same for all active banks, regardless of each bank’s own performance in the deposit market. Accordingly, the probability of adopting a specific pure strategy depends only on aggregate savings $\overline{S}$; i.e., for any active pair of banks $(l, h)$,

$$
\lambda^*_h(z^*_h | z_D, \lambda^*_h, q^*) = \lambda^*_l(z^*_l | \tilde{z}_D, \lambda^*_l \lambda^{*l|h}, q^*_{-l|h}),
$$

(50)

where $z_D$ and $\tilde{z}_D$ are any pair of deposit market outcomes that deliver the same active banks and the same amount of aggregate savings $\overline{S}$.

Assumption 1. [Weak Link between Sources and Uses of Funds] The institutional setup creates a weak link between sources and uses of funds.

Note that, if there is no externality (i.e., $\alpha = 1$), Assumption 1 is always satisfied. Before proceeding further, one more assumption is needed. It may be obvious, but it is assumed that banks are not discriminated against by firms. Let $z^{h/\setminus h}$ denote a set of pure strategies of banks in which bank $h$ and $l$ exchange their pure strategies.

Assumption 2. [No Discrimination Against Banks] In the competitive second stage, the best responses of firms to a set of banks’ strategies should not discriminate against any active banks. Specifically, a set of firms’ equilibrium strategies must satisfy the following: for any firm $j$ and for any active pair of banks $(l, h) \in D(z_D)^2$,

$$
q^*_j(z^*_j z^f_j | z^*_b, q^*_l) = q^*_j(z^*_j z^{h/\setminus h}_j, q^{*l|h}_j).
$$

(51)

Assumption 2 is not so restrictive because it states that only profit motives matter for firms to choose offers from banks. Thus, Assumption 2 is taken for granted in this section.\textsuperscript{34}

Under Assumptions 1 and 2, banks decide their loan market strategies conditional only on aggregate savings, not on their own fund positions, in an equilibrium. Indeed, using the Bayes rule, the equilibrium mixed strategies in the competitive second stage are defined as

$$
Q^*(z^b, z^f | \overline{S}) \equiv \lambda^*(z^b | z_D, q^*)q^*(z^f | z^b),
$$

(52)

with the deposit market outcome $z_D$ satisfying $\sum_{i=1}^I s_i = \overline{S}$.

\textsuperscript{34}If $j$-th firm faces the equally attractive offers from $h$-th and $l$-th banks but always prefers to accept $h$-th bank’s offer, then $j$-th firm is discriminating against $l$-th bank. Assumption 2 excludes such discrimination. This often leads to equal market share of banks offering the equally attractive offers, though picking one bank randomly is also consistent with Assumption 2 (and in this case, one bank would obtain 100 percent market share). Setting up a specific market sharing rule sometimes plays an important role in a discontinuous game. See discussions in Section IV.
Moreover, under Assumptions 1 and 2, expected revenues of bank $h$ and $l$ from loans are unchanged for the permutated strategy in which bank $h$ and $l$ exchange their loan market strategies and, accordingly, firms also exchange strategies for bank $h$ and $l$. This occurs because the joint probability of the permutated strategy is the same as that of the original equilibrium strategy:

$$Q^{*l\backslash h}(z^b, z^f | \overline{S}) = Q^*_{lj}(z^b, z^f | \overline{S}), \quad (53)$$

where $Q^*_{lj}(z^b, z^f | \overline{S})$ denotes $(h, j)$ element of $Q^*$ defined in (52) and $Q^{*l\backslash h}$ is the $(l, j)$ element of the permutated strategy,

$$Q^{*l\backslash h}(z^b, z^f | \overline{S}) \equiv \lambda^{*l\backslash h}(z^b | z_D, q^{*l\backslash h}) q^{*l\backslash h}(z^f | z_l, l\backslash h). \quad (54)$$

Equation (53) implies that loan market competition does not depend on results in the deposit market, and thus, banks do not have to worry about collecting too much deposit. Consequently, they compete aggressively in the deposit market, placing a restriction on a candidate for an equilibrium for the whole game. Proposition 1, below, shows that strategic tâtonnement satisfies Assumption 1 and hence condition (53). Then, Theorem 2, below, establishes that the equilibrium outcome is uniquely determined for the economy with strategic tâtonnement.

**Proposition 1.** Strategic tâtonnement is an institutional setup that satisfies Assumption 1 by creating a weak link between sources and uses of funds.

The proof is provided in online Appendix II. The basic idea is as follows: With strategic tâtonnement, the free-riding behavior is punished, and thus, limiting deposits to become a free-rider is not a profitable strategy. Indeed, there is no free-rider in an equilibrium. Then, the loan-market strategy does not depend on each bank’s collected deposits.

**Theorem 2.** [Unique Equilibrium Outcome] For any equilibrium strategies in the economy with strategic tâtonnement, equilibrium outcome is unique in terms of the deposit contract for each household and the loan contract for each firm, as specified in Theorem 1.

The proof is again provided in online Appendix II, but here I describe a sketch. Provided that banks do not worry about collecting too much deposit, the only force behind the deposit market strategies is each bank’s desire to become a monopoly lender. So, the deposit market rate is bid up to the monopoly loan rate $A$, and deposit amount is not restricted. Hence, there is a unique equilibrium deposit contract. In the loan market, banks need to allocate funds equally to firms and collect the loan rate $A$ to meet the obligations for depositors.

As it is clear from the proofs of Proposition 1 and Theorem 2, the important assumption is not strategic tâtonnement, per se, but the weak link condition. Indeed, strategic tâtonnement
is just an example of institutional setups that create weak link between sources and uses of funds. As such, both the existence of an equilibrium and the uniqueness of equilibrium outcomes can be established possibly with other institutional setups. In short, decentralized internalization of production externality can be more generally established with intermediaries.

IV. ECONOMY WITHOUT INTERMEDIARIES: NON-EXISTENCE OF A NASH EQUILIBRIUM

Now I show the second result of this paper: non-existence of an equilibrium in the economy without intermediaries (\( H = 0 \)). In this economy, firms have to finance their inputs by issuing bonds privately and directly to consumers. In this case, the capital market inherently suffers from theoretical difficulties studied in the discontinuous game literature. A key concept is a discontinuous payoff function. A firm’s profit suddenly changes when a firm becomes a monopolist. This discontinuity of the profit function in this economy is too severe to support any existence theorems of a Nash equilibrium.\(^{35}\)

Similar to the economy with banks, I consider a more general form of competition in which firms would choose both coupon rates and issue amounts of the bonds. In this regime, because the first-order conditions are essentially the same as in the Walrasian economy, only the Walrasian coupon rate \( \alpha A \) induces each firm to invest the same amount of capital \( k_j \) as the other firms and satisfies the fixed point condition \( k_j = K_j \). However, it cannot be an equilibrium interest rate, because a firm would be better off by offering a coupon rate a little higher than \( \alpha A \) and capturing all funds to become a monopolist, who can internalize externality and realize returns as much as \( A \). In other words, an interest rate lower than \( A \) cannot be an equilibrium, but at \( A \), all firms want to invest less than others, implying that zero investment is the only fixed point. Hence, no Nash equilibrium exists.

Note that, in the Walrasian economy, the setup follows Arrow and Debreu (1954), and the auctioneer is an independent agent who offers price only. The Walrasian equilibrium is a Nash equilibrium, because given the auctioneer’s action of setting price equal to \( \alpha A \), firms’ optimal strategy is to invest as much as others, \( k_j = K_j \), and there is no incentive for these price-taker firms to deviate from this strategy. Without the auctioneer, however, the Walrasian equilibrium is upset by a potential monopolist. Another way to support an equilibrium is to somehow prohibit the free-riding behavior, as effectively done by banks with strategic

\(^{35}\)A mixed Nash equilibrium always exists in a game with finite strategy space (Nash, 1950). However, if the strategy space is infinite (e.g., choosing investment amount from any real number between 0 and 1), existence of a Nash equilibrium depends on (dis)continuity of payoff functions. Dasgupta and Maskin (1986a, b) are seminal papers studying conditions for existence of a Nash equilibrium in games with discontinuous payoffs. Among others, Simon and Zame (1990) and Reny (1999) generalize and refine the literature.
\textit{tâtonnement} in the previous section. Then, a firm would like to invest as much as other firms, and the socially optimal equilibrium could prevail. However, without punishment, free riding is possible, and a deviant firm would invest less than others at the rate above the Walrasian interest rate.\footnote{This somewhat resembles “stability” of matches in the search literature in which agents search a good match in terms of flow utility but try to avoid quick separation because of (local) positive externality gains from a stable relationship. In particular, when on-the-job search is possible, multiple equilibria may arise (see a labor search model by Burdett, Imai, and Wright, 2004, and a marriage model by Cornelius, 2003). In a “bad” equilibrium, one partner continues to search for a better match while on the job and, with a rational belief, the other side does the same, resulting in a short-lived relationship. A “good” equilibrium would be selected if it is possible to block on-the-job search or extra-marital affairs. In this sense, it is similar in spirit to \textit{strategic \textit{tâtonnement}} that blocks free-riding behavior to support an equilibrium. Note, however, that the short-lived, “unstable,” match in the search literature is still an equilibrium phenomenon and a quite different concept from instability of a Nash equilibrium that I analyze here.}

This economy is \textit{isomorphic} to the economy with banks but without any interbank market that creates \textit{weak link between sources and uses of funds}. This is because, when the funding and lending are tightly connected, a free-riding bank obtains the marginal profit of a firm by picking a firm, tailoring its loan contract, and accordingly limiting the deposit amount. In other words, a bank faces essentially the same marginal profit condition as a firm faces in an economy without intermediaries.

\section{A. Institutional Setup with Private Direct Finance}

Firm $j$ posts a contract (i.e., a corporate bond) $z^f_{ij} \equiv (r^f_{ij}, s^f_{ij})$ to a consumer $i$. The contract consists of a coupon rate $r^f_{ij}$ and an issue amount $s^f_{ij}$. Each element can be taken from the nonnegative real number or left unspecified, abbreviated as \textit{N.S.} (not specified), so that $z^f_{ij} \in Z^f \equiv \{\mathbb{R}+ \cup \{\text{N.S.}\}\}^2$.

A consumer maximizes her life time utility by choosing savings amounts in each period. Consumer $i$’s strategy is denoted as $x^c_{ij} \equiv (r^c_{ij}, s^c_{ij})$. It is chosen from the strategy set $X \equiv \mathbb{R}_+^2$. However, this strategy set is constrained by firms’ strategies, $z^f_{ij}$. If a firm specifies the coupon rate of the bond, consumers cannot change it. I also assume that a consumer can always refuse to buy the bond, so that $s^c_{ij} = 0$ is always in the choice set. The constrained choice set of consumer $i$, given firm $j$’s offer $z^f_{ij}$, is then defined as:

\begin{equation}
G^c_{ij}(z^f_{ij}) \equiv \begin{cases} r^f_{ij} \times \mathbb{R}_+ & \text{if firm } j \text{ specifies } r^f_{ij} \text{ only,} \\ \mathbb{R}_+ \times (s^f_{ij} \cup \{0\}) & \text{if firm } j \text{ specifies } s^f_{ij} \text{ only, and} \\ r^f_{ij} \times (s^f_{ij} \cup \{0\}) & \text{if firm } j \text{ specifies both } r^f_{ij} \text{ and } s^f_{ij}. \end{cases}
\end{equation}

\footnote{I apologize for the abuse of notation, using similar or the same characters, which appeared in the previous sections, for slightly different concepts here and elsewhere in this section. Note that equity-type contracts, whose return depends on outcome, is not worth considering, as discussed in footnote 28.}
Note again that the choice set of the last case is either to “accept” \((r_{ijt}^F, s_{ijt}^F)\) or “reject” \((r_{ijt}^F, 0)\) the offer. Let \(z_i^F = (z_{i1}^F, z_{i2}^F, \ldots, z_{ij}^F)\) denote a vector of offers to consumer \(i\) from all firms. As before, the whole constrained choice set \(G_i^c(z_i^F)\) for a consumer \(i\) is defined as the Cartesian product of \(G_{ij}^c(z_{ij}^F)\) over all firms \(j \in J\). Given firms’ offers \(z_i^F\), consumer \(i\) chooses her strategy \(x_i \equiv \{x_{ij}\}_{j=1}^J\).

The feasible set for consumer \(i\), given particular offers from firms \(z_i^F\), is defined as a set of strategies \(x_i\), which must belong to the constrained choice set and satisfy the budget constraint,

\[
\Delta_{pi} = \Delta_p(m_i, z_i^F) \equiv \left\{ x_i : x_i \in G_i^c(z_i^F) \text{ and } \sum_{j=1}^J s_{ij}^c \in [0, m_i] \right\}. \tag{56}
\]

Note that the economy-wide resource constraint (9) is always satisfied, when a consumer \(i\)’s strategies are chosen from this feasible set. Also, note that the feasible set is nonempty and compact valued. Even when a consumer receives offers specifying an issue amount larger than her wealth level, she can still choose zero, which is in her budget set \(\Delta_{pi}\).

In equilibrium, consumers’ strategies \((r_{ij}^c, s_{ij}^c)\) will be realized, because consumers make final decisions given firms’ offers. The equilibrium “loan” amount becomes \(k_j = \sum_{i=1}^I r_{ij}^c s_{ij}^c\), and the average coupon rate is \(R_j = \sum_{i=1}^I r_{ij}^c s_{ij}^c / k_j\). Hence, a consumer’s second-period wealth \(M_i\) becomes a function of strategies of all the consumers \(x \equiv \{x_i\}_{i=1}^I\), given all firms’ offers \(z^F \equiv \{z_i^F\}_{i=1}^I\),

\[
M_i = \sum_{j=1}^J r_{ij}^c s_{ij}^c + w_i(\{k_j, R_j\}_{j=1}^J). \tag{57}
\]

Given wealth distribution \(m\), firms’ offer \(z^F\), and other consumers’ strategies \(x_{-i}\), consumer \(i\)’s problem is to choose her strategy \(x_i \in \Delta_{pi}\) to maximize her utility,

\[
\max_{x_i \in \Delta_{pi}} U(c_{i1}, c_{i2}) = u \left( m_i - \sum_{j}^J s_{ij}^c \right) + \beta u \left( \sum_{j=1}^J r_{ij}^c s_{ij}^c + w_i(\{k_j, R_j\}_{j=1}^J) \right). \tag{58}
\]

Similarly, a firm’s strategy is to maximize its profit by choosing its own bond offers \(z_{ij}^F\), given other firms strategies and consumers’ strategies.

**Definition 10.** The economy with private direct finance is the game \(\Gamma_P(m)\), given wealth distribution \(m \in \mathbb{R}_+^I\). It consists of \(I\) consumers and \(J\) firms, their strategy sets, and their

\[38\text{ Firm profit depends on coupon rate and issue amount. Again, a large number of consumers is assumed here, and each consumer’s share on firm ownership } \psi_{ij}^F \text{ is tiny so that the consumer’s own strategy cannot affect her profit income.} \]
utilities:
\[ \Gamma_P(m) \equiv ((I, J), (\Delta_{Pi}, Z^f), (U, \pi^f)), \]  

(59)

**Definition 11.** An equilibrium of a private direct finance economy is a set of Nash equilibrium strategies \((x^*, z^{f*})\) of the game \(\Gamma_P\).

**B. No Equilibrium Result**

**Proposition 2.** There exists no pure strategy Nash equilibrium in an economy with private direct finance.

*Proof.* Without loss of generality, I focus on the case with two firms. The following first-order condition determines the optimal loan amount that Firm 1 offers, with the associated interest rate \(r_1\): Given the investment level of Firm 2, \(k_2\),

\[ k_1 = \left( \frac{\alpha A}{r_1} \right)^{1/\alpha} k_2. \]  

(60)

Suppose the investment level is symmetric. Then, the fixed point condition says

\[ k_1 = k_2. \]  

(61)

The interest rate that satisfies both conditions is \(r_1 = r_2 = \alpha A\) only. Hence, in an equilibrium, if one exists, each firm must specify the loan amount equal to half of the aggregate Walrasian savings at interest rate \(\alpha A\),

\[ k_1 = k_2 = \frac{S(m, (\alpha A, N.S.))}{2}. \]  

(62)

Output of each firm is

\[ Ak_1 = Ak_2 = A \frac{S(m, (\alpha A, N.S.))}{2}, \]  

(63)

and profit is

\[ \pi^f_1 = \pi^f_2 = (A - \alpha A) \frac{S(m, (\alpha A, N.S.))}{2}. \]  

(64)

However, a firm has an incentive to deviate from this strategy by offering a slightly higher interest rate \(\alpha A + \epsilon\) with \(\epsilon > 0\) and specifying the savings amount equal to the aggregate savings under rate \(\epsilon A\).\(^{39}\) By doing so, the deviant firm becomes a monopolist and earns

\(^{39}\)Consumers will accept this offer: The first-period consumption is not optimized under \(\alpha A + \epsilon\) but still remains the same as before, and the second-period consumption is larger due to the higher interest rate.
higher profits than the profits under the fixed point interest rate $\alpha A$: for small $\epsilon$,

$$(A - (\alpha A + \epsilon)) S(m, (\alpha A, N.S.)) > (A - \alpha A) \frac{S(m, (\alpha A, N.S.))}{2}.$$  \hspace{1cm} (65)

Therefore, there is no symmetric pure strategy Nash equilibrium.

Assume now that the investment level is asymmetric, $k_1 > k_2$. The first-order conditions are

$$k_1 = \left(\frac{\alpha A}{r_2}\right)^{\frac{1}{1-\alpha}} k_2.$$  \hspace{1cm} (66)

and

$$k_2 = \left(\frac{\alpha A}{r_1}\right)^{\frac{1}{1-\alpha}} k_1.$$  \hspace{1cm} (67)

Combining the two first-order conditions (66) and (67),

$$k_1 = \left(\frac{\alpha A}{r_2}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha A}{r_1}\right)^{\frac{1}{1-\alpha}} k_1.$$  \hspace{1cm} (68)

This implies

$$r_1 r_2 = (\alpha A)^2.$$  \hspace{1cm} (69)

Let $(\hat{r}_1, \hat{k}_1)$, $(\hat{r}_2, \hat{k}_2)$ be the equilibrium strategies. Because $k_1 > k_2$, together with the two first-order conditions (66) and (67), the equilibrium condition (69) implies that

$$\hat{r}_1 < \alpha A < \hat{r}_2.$$  \hspace{1cm} (70)

There are two cases. In the first case, consumers may be indifferent between $z_{1f}$ and $z_{2f}$. Namely, a low interest rate with large savings may bring the same utility level as a high interest rate with small savings. In this case, however, the same logic in the symmetric investment case applies: Firm 1 will increase its coupon rate slightly and become the monopolist to earn higher profit. Hence, this case cannot be an equilibrium.

In the second case, either offer is strictly preferred to the other; for example, consumers prefer $z_{2f}$ to $z_{1f}$. In this case, $z_{1f}$ attracts the residual demand because $z_{2f}$ restricts the overall capital intake. Then, there is a room for Firm 2 to reduce its coupon rate slightly to $\hat{r}_2 - \epsilon$, for some $\epsilon > 0$, without failing to issue the same amount of bonds $\hat{k}_2$. Apparently, the smaller coupon rate lowers the cost and thus brings a larger profit to Firm 2. Hence, this case cannot be an equilibrium. In summary, there exists no asymmetric pure strategy Nash equilibrium.

Note that a “sharing rule” needs to be applied to allocate the savings across firms if consumers are indifferent among offered contracts. In most parts of the paper, explicit treatment of a sharing rule is not necessary but, here, it may be necessary to explain further.
Consider that there are two types of consumers and two firms. Suppose offers from two firms’ are equally attractive but Consumer 1 accepts only Firm 1’s offer and Consumer 2 accepts only Firm 2’s offer. This case represents the situation in which equally-attractive offers are accepted with different market shares.

First, it is impossible for firms to obtain monopolistic rents in a noncooperative way. If any monopolistic rents emerge from the Firm 1-Consumer 1 relationship, Firm 2 would take over Consumer 1’s savings by offering a slightly higher coupon rate with the same amount and thereby becomes the monopolist to earn higher profits.

Second, a sharing rule that assigns strictly positive shares to two firms cannot support an equilibrium. Suppose two equally attractive contracts obtain different positive market shares without monopolistic rents. With rational expectation, the equilibrium of this case, if any, can be rewritten equivalently with the contracts specifying the ex post loan amounts as the intended loan amounts. However, the case with two contracts with asymmetric loan amounts is already discussed above in which the market share has nothing to do with the proof.

Third, one exceptional case might exist with a sharing rule that assigns 100 percent share to one of two equally attractive contracts. However, such a sharing rule does not support an equilibrium, either.\footnote{In a pure strategy equilibrium, the market share must be assigned \textit{a priori}. However, the same proof applies to the case with a more general equilibrium in which the market share is assigned by public lottery. See discussions following Proposition 3.} An equilibrium contract with 100 percent share means that it should not specify the amount, N.S., or equivalently that it should specify the amount equal to the consumers’ willingness to supply, $s(m_i, r)$, at an equilibrium interest rate $r$.\footnote{If the prevailing contract restricts or over-recommends the savings amount, another firm can take advantage of the arbitrage opportunity.} The equilibrium interest rate cannot be lower than $A$; otherwise, another firm would offer a slightly higher, but less than $A$, coupon rate to take all the savings. Hence, essentially, $(A, N.S.)$ is the only equilibrium candidate. But again, even if the prevailing rate is $A$, a deviant firm would emerge, offer a slightly higher rate $A + \epsilon$ with a limited bond issue to invest less, free ride on others’ investments, and earn positive profits. This upsets the equilibrium candidate with a 100 percent share assignment as a tie-breaking rule. \textit{Q.E.D.}

For the mixed strategy case, as in the simple Bertrand competition, firms bid up from the lower end of the support of a mixed strategy, ending up with a degenerated pure strategy or negative profits. Or, similar to the Edgeworth cycle, without reaching a specific strategy, firms may revert back to a lower interest rate at some point and start bidding again. In particular, mixed strategies are difficult to survive under the standard macroeconomic assumptions of strict monotonicity and concavity of utility and (private) profit functions.
These characteristics assure that there is always another strategy superior to a given mixed strategy. The formal proof is provided in online Appendix II.

**Proposition 3.** There exists no mixed strategy Nash equilibrium in an economy with private direct finance.

Simon and Zame (1990) suggest that there always exists an equilibrium with an endogenous sharing rule under a general condition. More specifically, if a sharing rule for equally attractive contracts were chosen before competition, there would exist a sharing rule and associated allocation that together constitute an equilibrium. Although they call this sharing rule endogenous, it has to be exogenously given before firms compete. Yanelle (1998) uses Simon and Zame (1990)’s result and modifies it slightly: If public lotteries were available to select specific shares of consumers when firms set the same price, then there would exist an equilibrium in which one firm is assigned 100 percent market share and becomes a monopolist.

Simon and Zame’s mechanism or Yanelle’s version can be introduced here, but these techniques do not work here, as shown in the last paragraph of the proof of Proposition 2. It turns out that this game violates one of the conditions of Simon and Zame (1990): the upper hemicontinuity of the payoff correspondence. Similarly, this game violates Reny’s (1999) condition for the existence of a Nash equilibrium: better-reply security. Note that the root cause of the discontinuity of the payoff function is production externality, not strategic intermediation as in Yanelle (1998). Strategic intermediation adds another layer on top of the externality problem. More detailed discussions related to discontinuous game literature are provided in online Appendix III.

Note that introducing a similar mechanism of strategic tâtonnement among firms does not support an equilibrium. This is because sources and uses of funds are not separated, unlike the version for banks in Section III. Again, equilibrium coupon rate must be $A$, otherwise a monopolist firm emerges. But, at this rate, every firm wants to invest less than others. Here, firms have no incentive to coordinate to pay Pareto optimal rate $A$, which is the highest possible coupon rate in any equilibrium. This is because firms care about their own private marginal profits, while banks exploit firms and hence can be free from marginal pricing.

Also, introducing an equity-type contract does not work. Again, anything but less than $A$ cannot be the expected return of an equity-type contract because of the threat of a potential

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42Yanelle (1998), in a quite different model setup, restricts the domain by an upperbound, which is $A$ in the context of this paper. If assumed so in this paper, no interest rate would be allowed above $A$, and the economy analyzed here would also support an equilibrium at the bound. Apparently, this restriction is too ad hoc for the analysis here. However, note that the nonexistence result of this paper is not caused by the unbounded strategy space (i.e., interest rate and investment level can be any real number). Even if I restrict the strategy space to be some (sufficiently large) compact subset of real numbers, no Nash equilibrium exists.
monopolist. However, $A$ is the technologically highest possible equilibrium return. If there is a mixed strategy equilibrium in which some firms are probabilistically investing less than others, then overall expected return is less than $A$, due to concavity of the production function (as discussed in the proof of Proposition 3 in online Appendix II). In other words, overall expected return of an equity-type contract cannot exceed $A$. Then, the only possibility to have such an equity-type contract is the case that it returns $A$ in the equilibrium and less or more in the off-equilibrium. However, introducing this type of contract would make sense only if it can support an equilibrium. But, simply introducing an equity-type contract cannot restore an equilibrium here because, again, firms have no incentive to coordinate to pay the highest possible rate $A$ in an equilibrium.

V. CONCLUDING REMARKS

An economy with production externality is a key conceptual laboratory for many theoretical and empirical macroeconomic research, especially in the growth literature (see the review by Klenow and Rodríguez-Clare, 2005). I have found that such an economy is unstable without financial intermediaries in the sense that a Nash equilibrium does not exist. Using a game theoretic microfoundation, I have also confirmed that a Walrasian capital market, although it supports an equilibrium, does not bring the socially optimal allocation because of externality.

I have introduced strategic competition among banks in an economy with production externality. The strategic competition among banks turns out to support a Nash equilibrium. More importantly, it serves as a decentralized mechanism to internalize production externality and brings the Pareto optimal allocation. Specifically, banks compete for deposits to obtain potential monopoly profits, taking externality into account. Then, using loan contracts that specify both price and quantity, banks control firms’ investment decisions and implement the socially optimal capital allocation.

It was not straightforward to identify an equilibrium in the economy with banks, because the economy inherently lacks a Nash equilibrium without a further institutional setup. First, I have presented an example of institutions that establish an equilibrium. The example, strategic tâtonnement, is not far from reality: Banks should be able to negotiate loan terms (free recontracting) with firms and adjust their fund positions in an interbank market. This institutional setup can be viewed as an optimal mechanism to internalize production externality. Second, I have proven that the equilibrium outcome is uniquely determined regarding the deposit contract for each household and the loan contract for each firm under a general condition called a weak link of sources and uses of funds. This condition allowed banks to compete for deposits without worrying about their fund positions in the competitive
loan market.

In the equilibrium, each bank appears to form a firm group endogenously, as the equilibrium loan contract is exclusive (i.e., a firm borrows only from one bank). Then, each bank internalizes externality directly within a firm group and indirectly across firm groups. In other words, this paper successfully explains an origin of expansion of bank control and formation of competing firm groups, which are identified by various researchers as salient features of industrial development as well as of contemporary financial systems in many countries (e.g., Khanna and Yafeh, 2007). At the same time, such seemingly tightly connected business groups are sometimes accused of creating over-investment (from the viewpoint of private marginal product of capital) and crisis. This paper justifies over-investment from the viewpoint of the private marginal product of capital.

However, even with the proposed institutional setup, the equilibrium is fragile against equilibrium refinements (e.g., subgame perfection). Although this is not the case in the limit of infinite recontracting, the economy is not perfectly free from the intrinsic instability in this sense. Still, it is important to note that this instability is not brought by banks but deeply rooted in the economy with production externality. Moreover, this instability (i.e., possible non-existence of an equilibrium) is a deeper problem than the main focuses of the literature on bank competition and stability so far, which concentrate on either banks’ risk taking (in an equilibrium) or financial amplification due to financial frictions (on a dynamic equilibrium path).

Understanding banks’ roles in macroeconomic growth and stability has been a key interest in economics, at least since the Great Depression. Even with burgeoning literature after the recent financial crisis, none is yet to point out either deeply rooted instability in an economy with production externality or implications of having competing banks in such an economy. Therefore, the presented theory brings a new perspective and complements existing micro and macro theories of financial sectors based on informational problems and transaction costs. Moreover, the presented framework can readily serve as a microfoundation for future studies on linkages between a macroeconomy and its financial system, as it formulates

43 Although banks do not prohibit firms from acquiring other loans, a bank’s offer specifies a large enough loan amount for a firm to decline any other loan offers.

44 Bank control is one of the main issues when comparing financial systems (Allen and Gale, 2000) and when comparing corporate governance (Becht, Bolton, and Röell, 2003). Especially after the Asian Crisis (and the recent financial crisis), there are many papers criticizing “crony capitalism” (see empirical corporate governance papers, e.g., De Nicolo, Laeven, and Ueda, 2008). In a slightly different viewpoint, the quest for size of banks themselves and their client firms may well be a result of seeking monopoly rents from consumers (for example, see Rajan and Zingales, 2003). In the United States, these criticisms led to the introduction of the Glass-Steagall Act, which dissolved the bank-oriented financial system in the United States. But, this claim is also subject to counter argument; for example, Cantillo Simon (1998) reports bona fide values created by banks before the Glass-Steagall Act.
strategic behaviors of financial intermediaries interwoven with a standard macroeconomic model.
REFERENCES


APPENDIX I. WALRASIAN EQUILIBRIUM

Following Arrow and Debreu (1954), I consider a Walrasian equilibrium of the bond market as a game among consumers, firms, and an auctioneer ($H = 1$). The auctioneer’s strategy is to set the same interest rate for all savings returns and loan rates, $r = r_i = R_j$, $r \in \mathbb{R}_+$, so as to maximize her payoff, $\pi^a : \mathbb{R}_+^{1+I+J} \rightarrow \mathbb{R}_+$, which is the value of excess demand in the capital market:

$$\pi^a(r, \{s_i\}_{i=1}^I, \{k_j\}_{j=1}^J) = r \left( \sum_{j=1}^J k_j - \sum_{i=1}^I s_i \right). \quad (A1)$$

Note that the profit income of a consumer can be expressed as a function of the interest rate and loan amounts, $w_i(\{k_j\}_{j=1}^J, r)$, given the ownerships of firms and the auctioneer.

**Definition 12.** The Walrasian economy is the game $\Gamma_W(m)$, which is conditional on wealth distribution $m$ and consists of $I + J + 1$ agents ($I$ consumer, $J$ firms, and one auctioneer), strategy sets (savings, investments, and the interest rate), and utilities ($U, \pi^f$, and $\pi^a$):

$$\Gamma_W(m) \equiv ((I, J, 1), ([0, m_i], \mathbb{R}_+, \mathbb{R}_+), (U, \pi^f, \pi^a)). \quad (A2)$$

**Definition 13.** An equilibrium of the Walrasian economy is a set of Nash equilibrium strategies $(\{s_i^w\}_{i=1}^I, \{k_j^w\}_{j=1}^J, r^w)$ of the game $\Gamma_W(m)$ that satisfies economy-wide constraints (9) and (10).

The equilibrium is the same as in the case of the non-game formulation described by Romer (1986). This is because, given the auctioneer’s strategy of choosing an interest rate $r$, the first-order conditions of firms and consumers are the same as in Romer’s formulation.

**Proposition 4.** The Walrasian economy $\Gamma_W(m)$ has a unique equilibrium such that

(i) the return on investment is

$$r^w = \alpha A; \quad (A3)$$

(ii) the equilibrium savings is the consumer’s best response to the equilibrium interest rate,

$$s_i^w = \arg \max_{s_i} u(m_i - s_i) + \beta u(r^w s_i + w_i(\{k_j^w\}_{j=1}^J, r^w)); \quad \text{and} \quad (A4)$$

(iii) each firm invests an equal share of aggregate savings in the equilibrium

$$k_j^w = \sum_{i=1}^I s_i^w / J. \quad (A5)$$


The proof is standard and omitted, but the key points are the following: Given the offer of interest rate \( r \) from the auctioneer, each firm determines its investment based on its private marginal return \( \alpha AK_j^{1-\alpha} k_j^{\alpha-1} \), as it does not take into account the spillover effect on others; Due to the same interest rate, investment becomes symmetric, \( k_j = K_j \), and the equilibrium interest rate is determined at \( \alpha A \); Then, given this interest rate, consumers maximize the utility and determine the savings.

If the externality is internalized, the interest rate would be socially optimal \( A \), higher than the Walrasian interest rate, \( r^w = \alpha A \). This difference in the interest rate distorts the allocation of consumption over time in the Walrasian equilibrium away from the first best. For example, consider the utility function to exhibit the constant relative risk aversion, \( (1 - c)(1 - \sigma) / (1 - \sigma) \), which is often used in macroeconomic models. The Euler equation is

\[
\frac{c_2}{c_1} = \left(\beta r^w\right)^{\frac{1}{\sigma}}.
\]

(Appendix II)

(Appendix II) Proofs

A. Proof of Lemma 3

**Proof.** First, I show the \( T = \infty \) case. Since the same target contract \( z_{\phi} \) is adopted by all banks, it is the only contract offered to firms for all sessions \( \tau \). Knowing this, there is no gain in waiting. Hence, firms accept banks’ target contract \( z_{\phi} \) immediately.

The punishment contract \( z_w \) is necessary to sustain the above-mentioned strategy as an equilibrium. With this punishment, no bank wants to deviate from the target contract \( z_{\phi} \), because the revenue from firms would dwindle if it deviates. To see this clearly, consider a one-bank-and-one-firm deviation when \( \phi = 1 \). Bank 1 could think about offering Firm 1 a smaller amount of capital \( k \) to share the potential profits with the client firm by free riding on other firms’ investments. Note that this could be profitable, since \( AK_1^{1-\alpha} k^{\alpha-1} > 0 \) for \( k < K_1 \).

However, this scheme does not work under the proposed equilibrium strategy specified in Lemma 3. Other banks are offering the target contract \( (R = A, k_{lj} = S/J) \) to \( (J - 1) \) firms, and each firm tentatively accepts one of the offered contracts. Because each of \( (J - 1) \) firms borrows \( S/J \), total loans from \( H - 1 \) banks to \( (J - 1) \) firms is \( (J - 1)S/J \). Total deposits collected by \( (H - 1) \) banks is \( S - s_1 \). Hence, the net borrowing demand from the other
banks in the interbank market at interest rate $A$ is

$$
\sum_{t=2}^{H} B_t = \frac{J-1}{J} S - (S - s_1)
= s_1 - \frac{S}{J}.
$$

(A1)

This is the difference between the deposit amount that Bank 1 collected and the average size of deposits in the banking system. In other words, it is the residual demand for Bank 1’s fund in the interbank market. Depending on the size of the residual demand and Bank 1’s willingness to clear the interbank market, there are three scenarios.

1. If deposits in Bank 1 are equal to average deposits, $s_1 = \frac{S}{J}$, the residual demand/supply is equal to zero. In this case, there is no lending opportunity for Bank 1 in the interbank market. Then, it is optimal for Bank 1 to lend out the whole fund $s_1$ to firms, because otherwise some funds would be wasted. This is not a deviation.

2. If Bank 1 has deposits different from the average deposit per firm, $s_1 \neq \frac{S}{J}$, but decides to take up residual demand or supply in the interbank market, Bank 1 must lend the average size of all deposits in the banking system and lend or borrow funds equal to the difference between the average deposit and the deposit that Bank 1 collected. This is not a deviation either.

3. If Bank 1 has deposits different from the average deposit per firm, $s_1 \neq \frac{S}{J}$, and prevents the interbank market from clearing, then the following would occur.
   (i) Bank 1 would offer a loan contract that specifies the amount smaller than the average deposit per firm with a loan rate slightly higher than $A$.
   (ii) Because the interbank market does not clear, this deviation is detected by other banks. According to the specified strategy, punishment from other banks would be triggered. Thus, other banks would not confirm tentative loan offers, and the next session begins.
   (iii) In the second session and later sessions, other banks would offer the Walrasian contract $z_w = (R^b_{lj} = \alpha A, k^b_{lj} = N.S.)$ in the loan market with the associated interbank market contract $(\rho^b_{lj} = \alpha A, B^b_{lj} = \sum_{j=2}^{J} k^f_{r lj} - s_l)$. Here, if the deviant also offers the Walrasian contract, and if the offer is accepted by some firms, then the deviant bank would end up having revenue $\alpha A s_1$, lower than $\varphi A s_1$ that can be achieved by not

$^{45}$This is true even when Bank 1’s deposit is larger than the average deposit per firm. In this case, Bank 1 lends to the interbank market larger than the target strategy suggests, because Bank 1 can raise extra revenue per firm only by lending less than other banks.
deviating. However, if the deviant does not offer the Walrasian contract but a less attractive contract, then, given the specified strategy of other banks, firms do not accept the deviant’s offer but take other banks’ offers. In this case, the deviant is squeezed to lend $s_1$ to other banks at rate $\alpha A$ in the interbank market (otherwise, it would obtain zero revenue). Again, the revenue is lower than the one without deviating.

(v) Therefore, it is not profitable for Bank 1 to deviate from the specified strategy.\textsuperscript{46}

Other cases, such as one-bank-and-two-firm deviations, are analyzed similarly: Profitable deviation must specify the loan amount less than others, but the interbank market will not clear when some banks deviate and other banks stick to the target contract, $z_\phi$.

For $T < \infty$, the banks’ strategy specified in Lemma 3 constitutes a Nash equilibrium from the viewpoint of the beginning of the \textit{strategic tâtonnement}, or in the “normal form” game in which all sequences of strategies are presented at the outset. However, it is not robust for equilibrium refinement, such as subgame perfection. The only Nash equilibrium interest rate at the last session $T$ is $\alpha A$, as is the case with an economy with a one-shot loan market. Hence, when $T < \infty$, only the Walrasian contract survives through the \textit{strategic tâtonnement} as a subgame perfect equilibrium. This is because a part of the proposed strategy, “to keep offering the target strategy $z_{r-hj}^b = z_\phi$ as long as others did not deviate in the previous session $z_{r-1lj}^b = z_\phi$,” is not a credible promise.\textsuperscript{47}

If $T = \infty$, there is no “final” session, and thus, the proposed strategy (i.e., a set of the target contract, the punishment contract, and the associated interbank market contract) becomes always credible. Hence, the proposed strategy is a Nash equilibrium for any subgame, and thus, it is a subgame perfect equilibrium. \textit{Q.E.D.}

\textbf{B. Proof of Proposition 1}

\textit{Proof.} Lemma 3 clearly shows that there is a set of equilibrium strategies that satisfies Assumption 1: The bank’s equilibrium lending strategy does not depend on its deposit share. The remaining issue is whether all the equilibrium strategies in the economy with \textit{strategic tâtonnement} satisfy the \textit{weak link} condition (50).

\textsuperscript{46}Moreover, knowing that the prevailing interest rate is $\alpha A$ after the second session following a deviation, firms that received a deviant’s offer might not submit any demand to the deviant bank in the first session. This off-equilibrium strategy is consistent with the proposed equilibrium. However, without any banks’ deviation, no submission of demand by firms in the first session can open the next session but cannot trigger the banks’ punishment strategy. If firms keep refusing, firms end up having no capital and no revenue, according to the proposed equilibrium strategies of banks. Hence, firms accept the banks’ offers in the first session if no banks deviate from the equilibrium.

\textsuperscript{47}It is still a credible threat that banks revert to the Walrasian contract if some banks deviate.
The weak link condition implies that expected revenues from loans are the same for all banks regardless of their deposit market performances. This happens if no bank allows client firms to invest less than others. In other words, if the free-riding behavior is punished and thus prevented to happen, the weak link condition (50) will be satisfied. Therefore, it is sufficient to prove the following: If it constitutes a Nash equilibrium in the economy with strategic tâtonnement, then a set of strategies prevents free-riding behaviors.

I prove the contraposition: If it allows a bank to free ride on others, a set of strategies is not a Nash equilibrium. To show this, I use contradiction for a pure strategy equilibrium in a simple case with two banks and two firms. Suppose, in a Nash equilibrium, a bank free rides on others. In particular, assume that the pair of Bank 1 and Firm 1 free rides on the pair of Bank 2 and Firm 2. Bank 1 lends its deposits to finance Firm 1’s capital $k_1$, which is smaller than Firm 2’s capital $k_2$ financed by Bank 2’s deposits. Then, the highest possible return for Bank 2, $R_2$, is lower than $A$, 

$$\overline{R}_2k_2 = A \left( \frac{k_1}{k_2} \right)^{1-\alpha} k_2 < Ak_2.$$  \hfill (A2)

On the other hand, the highest possible return for Bank 1, $R_1$, is higher than $A$, 

$$\overline{R}_1k_1 = A \left( \frac{k_2}{k_1} \right)^{1-\alpha} k_1 > Ak_1.$$  \hfill (A3)

First, note that two banks must face zero profit in any supposed equilibrium. In other words, the deposit rate and the loan rate must be equal in an equilibrium: $r_1 = R_1$ and $r_2 = R_2$. If, instead, the Bank 1’s deposit rate is lower the loan rate, $r_1 < R_1$, Bank 2 can take over Bank 1’s business by offering the same loan contract $(R_1, k_1)$ as Bank 1 to Firm 1 and by offering almost the same deposit contract with a little higher deposit rate $r_1 + \epsilon$ with the same specified deposit amount for Bank 1’s depositors. And, with this strategy, Bank 2 would make a positive profit from this additional business. The same logic applies to the case with $r_2 < R_2$.\(^{48}\)

Second, given Bank 2’s strategy, $r_2 = R_2 < A$, Bank 1 has a profitable deviation: It would offer a contract with a slightly higher deposit rate than Bank 2, $(R_2 + \epsilon, N.S.)$, in the deposit market. Bank 1 would then become a monopolist and offer a loan contract $(A, S(R_2 + \epsilon, N.S.)/2)$ for each firm. Bank 1 would obtain a positive profit because of the positive spread between the loan and deposit rate, $A - (R_2 + \epsilon)$.

In summary, in the supposed equilibrium with free riding, the free-rider bank always

\(^{48}\)Note that deposit and loan amounts are the same ($s_h = k_h$) in an equilibrium. Banks can specify the amounts, and no slack, $s_h - k_h = 0$, is the most profitable given any deposit and loan rates; if there is a slack ($s_h - k_h > 0$), then by limiting the deposit intake, a bank makes a higher profit.
prefers the monopoly strategy (yielding positive profit) to any equilibrium free-riding strategies (yielding zero profit)—a contradiction. Therefore, if a set of pure strategies allows a bank to free ride on others, it is not a Nash equilibrium.

The proof in the case of the mixed strategy Nash equilibrium also runs similarly. Suppose there is a mixed Nash equilibrium in which a bank takes a free-riding strategy. Take any two pure strategies in the support of an equilibrium nondegenerated mixed strategy in the deposit market: the “small” strategy \((\tau, k)\) and the “big” strategy \((\tau, N.S.)\) with \(\tau > A > \tau\). Again, in an equilibrium, two banks earn zero profit because of the same reasoning as in the case of pure strategy Nash Equilibrium. There are four cases to consider:

1. When both banks choose “small” strategy, the associated probability is \(\lambda_1 s \lambda_2 s\) and the loan contracts are \((\tau, k)\) for both banks.

2. When Bank 1 chooses “small” strategy and Bank 2 chooses “big” strategy, the associated probability is \(\lambda_1 s \lambda_2 b\), and the loan contract is \((\tau, k)\) for Bank 1 and \((\tau, S(\tau, N.S.) - k)\) for Bank 2.

3. The opposite of Case 2 above happens.

4. When both banks choose “big” strategy, the associated probability is \(\lambda_1 b \lambda_2 b\), and the loan contracts are \((\tau, S(\tau, N.S.)/2)\) for both banks.

There is always a profitable deviation. For example, consider Bank 1’s alternative strategy to slightly increases the deposit rate offer in the “big” strategy to \((\tau + \epsilon, N.S.)\). While both banks’ loan contracts will not be affected in the first and second cases above, the loan contract outcomes change in the third and fourth cases. Given other contracts unchanged, Bank 1 will have negative profit as much as \(\epsilon(S(\tau, N.S.) - k)\) when the third case occurs. But, in the forth case, Bank 1 will become the monopolist and obtain the technologically highest return \(A\) from all firms. Thus, it will obtain a positive profit, as much as \((A - \tau - \epsilon)S(\tau + \epsilon, N.S.)\). The expected profit is positive for small \(\epsilon\), as its limit is strictly positive:

\[
\lim_{\epsilon \to 0} \lambda_1 b \lambda_2 s \epsilon(S(\tau, N.S.) - k) + \lambda_1 b \lambda_2 b(A - \tau - \epsilon)S(\tau + \epsilon, N.S.) = \lambda_1 b \lambda_2 b(A - \tau)S(\tau, N.S.) > 0.
\]

**Note:**

49 Bank 1 specifies deposit amounts while Bank 2 does not. Naturally, it is assumed that Bank 1 can always collect the specified amount of deposits and Bank 2 takes the residual supply. This is a specific assumption on the sharing rule, but the proof runs similarly with any other sharing rule assumptions.

50 The equal share is assumed here when both banks offer the same deposit contract. Again, the proof runs similarly with any other sharing rule assumptions.

51 If \(\epsilon\) is exactly equal to zero, then Bank 1 is not using any alternative strategy. Thus, this argument is only true for positive but small \(\epsilon\) (i.e., an alternative strategy in the neighborhood of the specified strategy).
The proof runs similarly for other any (possibly asymmetric) pair of pure strategies, or three or more pure strategies, in the support of an equilibrium mixed Nash strategy. Therefore, if a set of mixed strategies allows a bank to free ride on others and limit deposits, it is not an equilibrium. 

Q.E.D.

C. Proof of Theorem 2

The proof for Theorem 2 consists of six lemmas below. First, Lemma 4 shows that expected revenues from the loan market are the same for all banks. Second, Lemma 5 shows that the highest return is achieved when banks mimic the loan allocation by a monopoly lender. Third, Lemma 6 shows that banks do not discriminate against depositors, and there would be no arbitrage opportunities between the deposit and loan markets in an equilibrium. Fourth, Lemma 7 shows that competition in the deposit market drives up the deposit rate to $A$. Fifth, Lemma 8 shows that equilibrium loan contracts mimic the monopolist’s. Finally, Lemma 9 shows that banks compete for deposits essentially in price only.

Lemma 4. In the competitive second stage, under Assumptions 1 and 2, expected revenue in any Nash equilibrium is the same for all active banks regardless of performance in the deposit market.

Proof. This lemma is trivially true, if the equilibrium strategies are pure and symmetric and the associated market shares are the same. Indeed, Assumption 2 implies that, if the equilibrium strategies of banks and firms are pure and symmetric, market shares must be the same.

When the equilibrium strategies are pure but asymmetric, outcomes can be asymmetric. In this case, some banks free ride on others to achieve higher returns than others. However, condition (53) states that if bank $h$ adopted bank $l$’s strategy and bank $l$ adopted bank $h$’s, and if all firms exchanged their strategies toward bank $h$ with those toward bank $l$, then bank $h$ would earn bank $l$’s revenue, and bank $l$ would earn bank $h$’s revenue. Moreover, the probabilities of these two scenarios occurring (i.e., the original strategies and the exchanged ones) are the same in an equilibrium.

Hence, given an asymmetric equilibrium for bank $h$, any permutation of its equilibrium strategy with other banks, together with associated changes in firms’ strategies toward banks, would constitute an equilibrium. In other words, there exists a set of equilibria that is generated by permutating an equilibrium. Let it be called an equilibrium group generated by permutations of an equilibrium. For example, in a two-bank and two-firm economy, suppose a set of equilibrium strategies is expressed as

$$\{z^b, z^f\} = \{ (z_{11}^b, z_{12}^b), (z_{21}^b, z_{22}^b), (z_{11}^f, z_{12}^f), (z_{21}^f, z_{22}^f) \} = \{ (\gamma, \delta), (\xi, \psi), (\Gamma, \Delta), (\Xi, \Psi) \}.$$
Then, the following is also an equilibrium,
\[ \{ z^{b,2\downarrow 1}, z^{f,2\downarrow 1} \} = \{ (\xi, \psi), (\gamma, \delta), (\Xi, \Psi), (\Gamma, \Delta) \}. \]

These two equilibria constitute the equilibrium group generated by permutations of an equilibrium \( \{ (\gamma, \delta), (\xi, \psi), (\Gamma, \Delta), (\Xi, \Psi) \} \). This notion is expressed similarly using mixed strategies; for example, by defining that \( \lambda_{11} \) has a probability mass of one at \( \gamma \) and zero elsewhere, that \( \lambda_{12} \) has a probability mass of one at \( \delta \) and zero elsewhere, and so on.

If \#D banks are active, there are only \#D numbers of equilibrium outcomes for a particular bank in the equilibrium group generated by an equilibrium. This is because bank \( h \)'s outcome is determined only by bank \( h \)'s strategy, \( (z_{bh1}, z_{bh2}, \ldots) \), and firms’ strategies toward bank \( h \), \( (z_{fh1}, z_{fh2}, \ldots) \). Hence, exchanging strategies among bank-firm pairs that do not involve bank \( h \) would not affect bank \( h \)'s profits.

Given an equilibrium strategy group, condition (53) assures that a bank faces the same chances to realize any outcomes among \#D possibilities.\(^{52}\) If the equilibrium strategy group is generated by permutations of an equilibrium that gives outcome \( (R^*_{hj}, k^*_{hj}) \) to \((h, j)\) bank-firm pair, then, for all active banks, the expected revenue is the same,

\[
\frac{1}{\#D} \sum_{h \in D(z_D)} \sum_{j=1}^{J} R^*_{hj} k^*_{hj}, \tag{A5}
\]

which is the average of revenues from all strategies in the equilibrium strategy group.

Moreover, multiple equilibrium may exist, and accordingly multiple equilibrium strategy groups may exist. In this case, condition (53) implies that all active banks face the same probability of selecting a specific equilibrium strategy vector and, thus, a specific equilibrium strategy group. Therefore, expected revenue in the competitive second stage is just a linear combination of (A5) and is the same for all active banks. Let \( e_1, e_2, \ldots, e_N \) denote equilibrium strategy groups and \( \Psi(e_n) \) denote equilibrium probability of realization of the \( n \)-th equilibrium strategy group. The expected revenues are again the same for all active banks,

\[
\sum_{n=1}^{N} \left( \frac{1}{\#D} \sum_{h \in D(z_D)} \sum_{j=1}^{J} R^*_{hj} k^*_{hj} \right) \Psi(e_n), \tag{A6}
\]

\(^{52}\)These \#D equilibrium outcomes may be the same, particularly if an original equilibrium is pure and symmetric. For the sake of simplicity, I treat each equilibrium generated by a permutation of the original equilibrium as a distinct equilibrium to be counted towards \#D outcomes.
and so are the expected returns,

\[ R^e \equiv \sum_{n=1}^{N} \left( \frac{1}{\#D} \sum_{h \in D(z_D)} \sum_{j=1}^{J} R_{hj}^* k_{hj}^* \right) \Psi(e_n). \]  

(A7)

**Q.E.D.**

**Lemma 5.** A bank’s expected revenue is the highest if active banks together mimic a loan allocation assigned by a monopolist. In this case, expected return \( R^e \) is equal to \( A \), the technologically highest return under symmetric investment.

**Proof.** Lemma 4 shows that all active banks face the same expected revenue, given an equilibrium strategy group. But, as implied by Lemma 1, the sum of profit income is the largest when the same amount of capital is allocated among firms as if done by a monopolist lender. Because all banks have the same revenue \( R^e \) (Lemma 4), for any equilibrium group,

\[ R^e = \frac{1}{\#D} \sum_{h \in D(z_D)} \sum_{j=1}^{J} R_{hj}^* k_{hj}^* \leq \frac{1}{\#D} \sum_{j=1}^{J} AK_{Mj}^{1-\alpha} k_{Mj}^\alpha = \frac{1}{\#D} Ak_{Mj}. \]  

(A8)

When asymmetric multiple equilibrium strategy groups exist, (A8) is true for each equilibrium strategy group, and overall expected revenue is a linear combination of (A8). Hence, it has the same upper bound:

\[ \sum_{n=1}^{N} \left( \frac{1}{\#D} \sum_{h \in D(z_D)} \sum_{j=1}^{J} R_{hj}^* k_{hj}^* \right) \Psi(e_n) \leq \sum_{n=1}^{N} \left( \frac{1}{\#D} \sum_{j=1}^{J} AK_{Mj}^{1-\alpha} k_{Mj}^\alpha \right) \Psi(e_n) = \frac{1}{\#D} Ak_{Mj}. \]  

(A9)

Therefore,

\[ R^e \leq A. \]  

(A10)

**Q.E.D.**

**Lemma 6.** Equilibrium deposit rates of a bank are nondiscriminatory among depositors; that is, \( r_{hi} = r_h \), for all \( i = 1, 2, \ldots, I \). Moreover, a no-arbitrage condition holds in an equilibrium: \( r_h = r = R^e \).

**Proof.** There are two claims in this proof. The first claim is that deposit rates may vary among depositors, but, in an equilibrium, the weighted average of deposit rates must be equal to the expected return,

\[ \frac{\sum_{i=1}^{I} r_{hi}^* s_{hi}^*}{\sum_{i=1}^{I} s_{hi}^*} = R^e, \]  

(A11)

in a pure strategy equilibrium and in the support of a mixed equilibrium.
Here is the proof for this first claim. If the left-hand side of (A11) is strictly greater than the right-hand side, bank $h$ would have negative profit and cease to operate. If the left-hand side is strictly smaller than the right-hand side, an apparent arbitrage opportunity by rival banks would exist, offering slightly higher interest $r_{hi} + \epsilon$ for a depositor. With this strategy, it would be feasible for a rival bank to collect the same deposit amount as bank $h$ and to drive out bank $h$ from the competition.\footnote{With a higher interest rate, this specified deposit amount might not be the optimal amount for households. However, any household would prefer this deviant’s offer, because the deposit rate is higher for the same amount of deposits.} And, the rival bank would expect positive profits. Therefore, (A11) must hold in an equilibrium.

The second claim is that banks would not discriminate against depositors. Proof is given by contradiction. Assume that bank $h$ discriminates against some depositors in an equilibrium. Condition (A11) then implies that some depositors are offered higher-than-average interest $r_{hi} > R^e$ and at least one of the others, say the $m$-th consumer, faces an offer with lower-than-average interest $r_{hm} < R^e$. Here, however, a rival bank has an arbitrage opportunity, something impossible in an equilibrium. To see the arbitrage opportunity, note that a rival bank can earn positive profits by offering a slightly higher deposit rate $r_{hm} + \epsilon$ to the $m$-th consumer and specifying the same deposit amount. Since the expected loan rate is the same, the profit from $m$-th consumer is positive: $(R^e - (r_{mh} + \epsilon))s_{mh} > 0$.

In summary, deposit rates offered by a bank must be nondiscriminating among depositors in an equilibrium. Hence, $i$ subscript of $r_{hi}$ can be omitted as $r_h$. But, condition (A11) implies $r_h = R^e$ and, thus, the deposit rate must be the same for all active banks in an equilibrium. Accordingly, subscript $h$ can be also dropped, so that $r_h = r = R^e$. \textit{Q.E.D.}

The next two lemmas show that banks bid up their deposit rates until $r = A$, in an attempt to capture monopoly profits, and that banks have to charge at least this rate in the loan market to meet their own nonnegative profit condition.

**Lemma 7.** The deposit rate $r$ is equal to $A$ in a Nash equilibrium of the whole game.

**Proof.** Lemma 6 shows that an equilibrium deposit rate is the same for all banks and depositors. Lemmas 5 and 6 implies $r = R^e \leq A$. However, a deposit rate $r = A < A$ cannot be an equilibrium rate. If such a deposit rate prevailed in an equilibrium, a bank could become a monopolist by deviating to offer a slightly higher deposit rate $A + \epsilon$ with specifying the Walrasian savings amount under rate $A$. The deviant bank then earns a positive profit $(A - A - \epsilon)S(m, (A, N.S.))$. There always exists $\epsilon > 0$ for some banks to make this deviation profitable if $A < A$ prevails.

To clarify this, let us consider two cases, assuming the prevailing deposit rate is $A < A$. First, consider a case in which some banks do not collect any deposits. If one of these
unsuccessful banks deviates and offers a slightly higher deposit rate \( A + \epsilon \), it instantly increases its profit from zero to \((A - A + \epsilon)S(m, (A, N.S.))\). Obviously, any \( \epsilon < A - A \) works well.

Second, consider a case in which all banks collect some deposits. Let \( \gamma_h \in (0, 1) \) denote the deposit market share of bank \( h \). For deviation to be profitable, profit without deviation must be less than the profit with deviation. Since the expected loan rate is \( R^e \leq A \), profit without deviation has the natural upperbound,

\[
\gamma_h(R^e - A)S(m, (A, N.S.)) \leq \gamma_h(A - A)S(m, (A, N.S.)). \tag{A12}
\]

Here, take \( \epsilon \equiv (A - A) - \gamma_h(A - A) - \delta \) with some small \( \delta > 0 \). Because \( 0 < \gamma_h < 1 \) (all banks collect some deposits), there always exists such \( \epsilon > 0 \). Using this \( \epsilon \), it becomes clear that the right-hand side of (A12), the upperbound of profit without deviation, is smaller than the profit from the deviation,

\[
\gamma_h(A - A)S(m, (A, N.S.)) < (A - A - \epsilon)S(m, (A, N.S.)). \tag{A13}
\]

Q.E.D.

Lemma 6 \((R^e = r)\) and Lemma 7 \((r = A)\) imply that, in an equilibrium, the expected loan rate must be equal to the deposit rate determined by competition in the deposit market:

\[
R^e = r = A. \tag{A14}
\]

For all banks to achieve the expected loan rate \( A \), the only possible way turns out to mimic the social planner’s loan allocation. This leads to the next Lemma.

**Lemma 8.** If an equilibrium exists, equilibrium loan contracts generate a Pareto-optimal allocation. More specifically, each firm faces only one type of offer, which is \((A, S(m, (A, N.S.))/J)\), and accepts one bank’s offer, while rejecting offers from other banks. Accordingly, each firm invests the same amount \( S(m, (A, N.S.))/J \) and repays them with gross loan rate \( A \).

**Proof.** It is easy to show that the aggregate production is uniquely maximized by symmetric capital allocation among firms given any total funds.\(^{54}\) Thus, symmetric capital allocation among firms is necessary for all banks to achieve the expected loan rate to be the socially

\(^{54}\)A formal proof is provided in an earlier version of this paper.
best return, $R^e = A$. Therefore, each firm invests the equal fraction of the total savings,

$$\sum_{h=1}^{H} k_{hj} = \frac{S(m, (A, N.S.))}{J} = K. \quad (A15)$$

Because banks’ expected return is $R^e = A$, the weighted average loan rate must be $A$; that is,

$$\frac{\sum_{h=1}^{H} R_{hj}k_{hj}}{\sum_{h=1}^{H} k_{hj}} = \frac{\sum_{h=1}^{H} R_{hj}k_{hj}}{S(m, (A, N.S.))/J} = A. \quad (A16)$$

Note that the two conditions above, (A15) and (A16), do not exclude the possibility that loan contracts are different among bank-firm pairs in an equilibrium. However, this is not the case.

To clarify this, I first show that only one loan rate prevails in an equilibrium by contradiction. Then, I show that all banks specify the same amount of loans.

Suppose that different loan rates exist in an equilibrium. Then, at least one bank must offer a loan rate greater than $A$ to firm $j$, as (A16) describes that the weighted average of equilibrium loan rates must be $A$. Sorting banks according to the loan rate to firm $j$ from low to high, the last $H$-th bank is assumed to offer the highest loan rate $R_H > A$ without loss of generality. Because bank $H$’s offer is accepted by a firm in an equilibrium, the marginal cost for the firm of accepting the offer must be lower than or equal to the marginal revenue. The marginal cost is $R_H k_H$. The marginal revenue is the difference between investing the equilibrium amount $K$, which must be the same for every firm as shown in (A15), and the same equilibrium amount but without the last bank’s fund $k_H \leq K$:

$$AK^{1-\alpha}K^\alpha - AK^{1-\alpha}(K - k_H)^\alpha. \quad (A17)$$

The marginal revenue minus the marginal cost, then, is

$$AK^{1-\alpha}K^\alpha - AK^{1-\alpha}(K - k_H)^\alpha - R_H k_H,$$

$$< AK - AK^{1-\alpha}(K - k_H)^\alpha - Ak_H, \quad (A18)$$

$$= A(K - k_H) - AK^{1-\alpha}(K - k_H)^\alpha.$$

However, the first line of (A18) is always strictly negative, because, in the last line of (A18), the first term is less than or equal to the second term, as is easily shown:

$$A(K - k_H) \leq AK^{1-\alpha}(K - k_H)^\alpha,$$

$$1 \leq \left( \frac{K}{K - k_H} \right)^{1-\alpha}. \quad (A19)$$
Obviously, a nonspecified option for the loan amount cannot alter this result. Different loan rates, therefore, cannot exist in an equilibrium.

It is now clear that loan rates are the same \( A \) for any bank-firm pairs in an equilibrium. At this loan rate, firms want to borrow less than others. Hence, a firm will pick a loan contract that specifies the least loan amount and reject all other offers. But, in an equilibrium, as shown above, (A15) must be satisfied, and thus, the least amount of loan offered to a firm must be the Pareto-optimal amount, \( S(m, (A, N.S.))/J \). In an equilibrium, it must also be the case that the sum of loans over \( J \) firms is equal to \( S(m, (A, N.S.)) \) in order to clear the loan market. Therefore, symmetrical loan amount \( S(m, (A, N.S.))/J \), together with loan rate \( A \), is the only equilibrium offer by each bank to each firm. \( Q.E.D. \)

**Lemma 9.** In an equilibrium, a depositor faces at least one unspecified offer, or, if all are specified, she must be able to combine offers to replicate her willingness to supply capital at deposit rate \( A \). For a bank, not specifying deposit amounts \( s_{hi}^b = \{N.S.\} \) and specifying the depositor’s willingness to supply \( s_{hi} = s(m_i, (A, N.S.)) \) are two dominant strategies. Other specifications can be Nash equilibrium strategies as long as the combination of offers from several banks replicates each depositor’s willingness to supply capital at deposit rate \( A \).

**Proof.** If any combination of offers does not fulfill each depositor’s willingness to supply at deposit rate \( A \), then an apparent arbitrage opportunity exists so that these offers cannot constitute an equilibrium. Indeed, if any combination of offers limits a savings amount less than a depositor’s willingness to supply at deposit rate \( A \), a bank can deviate to offer a lower rate \( A - \epsilon \) at which the deviant bank is still able to collect all the specified deposit amount as before. Using this strategy, this bank would earn extra profit, because the equilibrium expected loan return is \( A \) by Lemma 8. On the other hand, if any offers from banks specify a savings amount larger than a depositor’s willingness to supply at \( A \), a bank will deviate to offer a nonspecified amount with a slightly lower interest rate \( (r_h = A - \epsilon, s_h = \{N.S.\}) \) and could capture the entire deposit market and earn the monopoly rent.

Therefore, in an equilibrium, at least one combination of offers must give each depositor exactly the same as her willingness to supply capital at deposit rate \( A \). Because bank’s strategies \((A, N.S.)\) and \((A, s(m_i, (A, N.S.)))\) always satisfy these conditions, these two strategies are always equilibrium strategies. Note that, since these equilibrium strategies are not conditional on other banks’ (as well as consumers’ and firms’) strategies, they are weakly dominant strategies.

However, any specified amounts with deposit rate \( A \) may be a Nash equilibrium conditional on other banks’ strategies, because only a combination of offers concerns depositors. For example, when some banks offer \((A, s(m_i, (A, N.S.))/3)\) to a depositor, other banks’ strategy \((A, 2s(m_i, (A, N.S.))/3))\) to the same depositor is an equilibrium
strategy, as these strategies enable the depositor to make deposits equal to her willingness to save, $s(m_i, (A, N.S.))$.

Q.E.D.

D. Proof of Proposition 3

Proof. I use contradiction: If a nondegenerate mixed strategy is an equilibrium strategy for a firm, then I will show that there always exists another strategy that brings a higher profit. Without loss of generality, I focus on the case of two firms.

Assume there exists a nondegenerate mixed strategy equilibrium. Let $q^*_1$ denote a nondegenerate equilibrium mixed strategy of Firm 1 and $q^*_2$ denote Firm 2’s equilibrium mixed strategy (possibly degenerated). Note that for $q^*_1$ to be an equilibrium, given $q^*_2$, any pure strategies in the support of $q^*_1$ should provide the same expected profit to Firm 1.\textsuperscript{55}

Hence, without loss of generality, analysis here focuses on any arbitrary two points in the support of Firm 1’s equilibrium mixed strategy: $z^f_{1a} = (r_{1a}, k_{1a})$ and $z^f_{1b} = (r_{1b}, k_{1b})$. Both should lie on the same iso-profit curve (IPC), defined on $(k, r)$-plane (see Figure 1).\textsuperscript{56}

Consider an equilibrium where Firm 1 adopts a nondegenerate mixed strategy $q^*_1$ but Firm 2 offers a pure strategy $z^f_2 = (r_2, k_2)$. Take any arbitrary two points in the support of Firm 1’s equilibrium mixed strategy, $z^f_{1a} = (r_{1a}, k_{1a})$ and $z^f_{1b} = (r_{1b}, k_{1b})$, with associated

\textsuperscript{55}See a textbook, for example, Osborne and Rubinstein (1994, pp. 33-34)

\textsuperscript{56}Figure 1 shows the case of $r_{1a} < r_{1b}$ and $k_{1a} < k_{1b}$, but the same analysis applies to the case with $r_{1a} < r_{1b}$ and $k_{1a} > k_{1b}$. 
equilibrium probability $q_{1a}^*$ and $q_{1b}^*$, respectively. For them to be in the support of an equilibrium mixed strategy, both $z_{1a}^f$ and $z_{1b}^f$ must face positive demand: Firm 1’s offer is preferred by consumers to Firm 2’s strategy $z_2^*$. Or, consumers are still eager to buy Firm 1’s bonds after purchasing all bonds issued by Firm 2 (i.e., positive residual demand).

Given Firm 2’s strategy $z_2^*$, Firm 1’s expected profit from these two pure strategies in the support of the mixed strategy is a convex combination of underlying profits:

$$E_{1ab}^* \pi \equiv Q_{1a}^* \pi(k_{1a}, k_{2}, r_{1a}) + Q_{1b}^* \pi(k_{1b}, k_{2}, r_{1b}),$$  \hspace{1cm} (A20)

where weights are conditional probabilities given by the equilibrium mixed strategies, $Q_{1a}^* \equiv q_{1a}^*/(q_{1a}^* + q_{1b}^*)$ and $Q_{1b}^* = 1 - Q_{1a}^*$.

Define $\bar{z}_f = (\bar{r}, \bar{k})$ as a weighted average of $z_{1a}^f$ and $z_{1b}^f$ with $Q_{1a}^*$ and $Q_{1b}^*$ as the weights. This $\bar{z}_f$ also faces positive demand. To see this, consider consumers’ indifference curves (IDC) on $(k, r)$ plane (see Figure 1). Since a consumer prefers a higher coupon rate given her savings $k$, the utility level increases with $r$ as long as Firm 1’s offers are preferred to Firm 2’s offer $z_2^f$. Given the coupon rate $r$, the consumer’s maximization problem determines a unique optimal savings level $k = S(m, (r, N.S.)).$ Hence, given a utility level, an indifference curve decreases toward the optimal $k$ and then increases for excessive $k$, because consumers demand a higher interest rate to compensate for nonoptimal savings $k$. In Figure 1, note also that the utility level is higher on the upper indifference curve (IDC) than the lower one (IDC’).

Because any two indifference curves, corresponding to different utility levels, do not cross each other, the utility level of the weighted average offer $\bar{z}_f$ is strictly preferred to either $z_{1a}^f$ or $z_{1b}^f$, whichever is less attractive to consumers. However, as even less attractive offers face positive demand from consumers in an equilibrium, the average offer $\bar{z}_f^f$ must attract positive demand at least as much as the less attractive offers face.\footnote{In figure 1, $z_{1a}^f$ is less attractive than $z_{1b}^f$ and also than $\bar{z}_f^f$.}

While $z_{1a}^f$ and $z_{1b}^f$ lie on the same iso-profit curve, the weighted average offer $\bar{z}_f^f$ lies inside the iso-profit curve (IPC). Indeed, it is easy to show that the iso-profit curves on the $(k, r)$-plane is strictly concave and that it has a single peak (i.e., the unique optimal capital level exists, given a coupon rate). Also, profit is higher on the lower iso-profit curve (IPC’) than the upper one (IPC).

Therefore, Firm 1 can obtain a strictly higher profit by rearranging its mixed strategy: removing some positive probability mass on $z_{1a}^f$ and reallocating it on $\bar{z}_f^f$. In other words, there always exists another mixed strategy that strictly dominates an equilibrium nondegenerate mixed strategy. This is a contradiction. Note that almost the same proof applies to the case in which both Firm 1 and Firm 2 adopt nondegenerate mixed
There is no theorem that assures the existence of a Nash equilibrium in an economy without intermediaries or an auctioneer. This is because the payoff function is seriously discontinuous.

Simon and Zame (1990) suggest that there always exists an equilibrium with an endogenous sharing rule under a general condition. However, the economy without intermediaries violates one of the conditions of Simon and Zame (1990), namely, the upper hemicontinuity of the payoff correspondence.

Without loss of generality, consider the case with two firms, \( j = 1, 2 \), and assume a large number of (measure one) households with identical initial wealth and firm ownership. I focus on a neighborhood of the strategy with which Firm 1 tries to become a free rider and Firm 2 tries to become a monopolist at an interest rate \( A \) by not specifying the issue amount; i.e.,

\[
\hat{z}_1^f = (A, \underline{s}) \text{ and } \hat{z}_2^f = (A, N.S.),
\]

where \( \underline{s} \) is assumed to be less than the privately optimal capital investment by Firm 1 when Firm 1 sells out all the bonds.

Note that, given coupon rate \( A \), the privately optimal capital investment by Firm 1 is \( k_1 = \alpha^{1/(1-\alpha)}k_2 \). In this case, the capital investment by Firm 2 becomes \( k_2 = S(m, (A, N.S.))/(1 + \alpha^{1/(1-\alpha)}) \). Hence, for example, if \( \alpha = 1/2 \), then \( \underline{s} < S(m, (A, N.S.))/5 \) is the assumption.\(^{58}\)

Because both bonds bear the same coupon rate \( A \), personal savings is always equal to \( s(m, (A, N.S.)) \). However, allocation of saving between two firms can be different. For example, a consumer may choose to invest all her savings into Firm 2’s corporate bonds, or she may invest in Firm 1 up to \( \underline{s} \) and then the rest (the residual demand) in Firm 2. In sum, the market shares and profits of two firms depend on the sharing rule. When Firm 1 has a positive share \( \nu \in (0, 1] \), its profit can be written as

\[
\pi_1 = Ak_2^{1-\alpha}k_1^\alpha - Ak_1 = A(S(m, (A, N.S.)) - \nu \underline{s})^{1-\alpha} (\nu \underline{s})^\alpha - A\nu \underline{s} \geq 0.
\]

Figure 2 illustrates Firm 1’s profit in the \( k_1^f \)-dimension of the neighborhood of the strategy \( \hat{z}^f = (\hat{z}_1^f, \hat{z}_2^f) \). When Firm 1 offers \( k_1^f = \underline{s} \), the largest possible profit for Firm 1 can be achieved when it sells out all the bonds and raises the maximum amount of capital \( k_1 = \underline{s} \).

\(^{58}\)The socially optimal capital investment is always the equal investment from two firms, \( S(m, (A, N.S.))/2 \).
Because the optimal size is still larger by assumption.\(^{59}\) Because Firm 1 restricts the bond issues, there may exist positive residual supply of funds (i.e., positive residual demand for bonds), which goes to Firm 2. As less bonds are purchased by households (i.e., \(k_1 = \nu s\) with \(\nu\) declining), the Firm 1’s profit would shrink to zero. Zero profit is the smallest possible profit, which would realize with zero market share.

In the right region of \(s\) on the \(k_1^f\)-dimension, Firm 1 offers a contract specifying the amount a little more, \(k_1^f > s\), while keeping the interest at the same \(A\). Still, the offered amount is assumed to be less than the privately optimal investment in the neighborhood of \(\hat{z}^f\). Thus, the analysis is the same as at the strategy \(\hat{z}^f\). The largest possible profit is larger than at \(k_1^f = s\), because the size becomes closer to the optimal. In the left neighborhood of \(\hat{z}^f\), the situation is similar, although the largest possible profit under the maximum share is smaller than at \(k_1^f = s\).

Figure 3 illustrates Firm 2’s profit in the \(k_1^f\)-dimension of the neighborhood of the strategy \(\hat{z}^f\). The largest possible profit for Firm 2 is zero, when Firm 1 has zero market share. This is because the monopoly output is \(AS(m, (A, N.S.))\), but the cost of capital is also \(AS(m, (A, N.S.))\).

When Firm 1 has some positive share, the profit of Firm 2 can be written as

\[
\pi_2^f = A (\nu s)^{1-\alpha} (S(m, (A, N.S.)) - \nu s) - A (S(m, (A, N.S.)) - \nu s).
\]  
(A3)

\(^{59}\)As before, variables without superscript are equilibrium values.
If Firm 2 has a positive share but not the full share, then the maximum profit will be obtained when Firm 1 (the free rider) invests most, \( s \), under given strategy \( \hat{z}^f \). Still, even at this maximum, Firm 2’s profit is strictly less than zero. As Firm 1’s share declines, Firm 2’s profit declines and approaches \( -AS(m, (A, N.S.)) \). However, it never takes the value \( -AS(m, (A, N.S.)) \), because, as discussed, Firm 2’s profit is zero under the full share of Firm 2 (the zero share of Firm 1). This means that, when its share changes, Firm 2’s profit changes discontinuously, and the graph is not closed.

The situation is the same in the right and left neighborhoods of the point \( k_1^f = s \). The middle line is increasing in \( k_1^f \), as there is less free riding when Firm 1 sells out the all bonds.

The sum of two firms’ profits (Figure 4) inherits the shape of Figure 3. To see the (dis)continuity of the sum of the two firms’ profits at strategy \( \hat{z}^f \), it is necessary to examine how their profits vary with a slight change in each element of strategy \( \hat{z}^f \). I will show, below, that it is not upper hemicontinuous in quantity \( k_1^f \).

Suppose Firm 1’s share, and thus investment \( k_1 \), declines toward zero at the strategy \( \hat{z}^f \). The sum of two firms’ profits declines toward \( -AS(m, (A, N.S.)) \), but it never takes that value. When Firm 1’s share is exactly equal to zero, the sum of two firms’ profits suddenly becomes zero. This occurs because Firm 2 becomes a monopolist and its profit is zero. If

\[ \nu = 0 \text{ (zero share for Firm 1)} \]

\[ \nu = 1 \text{ (maximum share for Firm 1)} \]

\[ \nu \to 0 \]
both firms have a positive share, the sum of profits is strictly less than zero. This is because the aggregate revenue is strictly less than that of a monopolist, due to free riding by Firm 1. The graph is open at $\pi_1 + \pi_2 = -AS(m, (A, N.S.))$ for any $k_1^f$ in the neighborhood of $\hat{z}^f$. Thus, the payoff correspondence is not upper hemicontinuous.

I would also like to note the conditions that appeared in Dasgupta and Maskin (1986a) to better understand the situation of discontinuity of payoff functions in this paper. They analyze a discontinuous game based on a single-valued payoff function. In particular, Dasgupta and Maskin (1986b) discuss the Rothschild-Stiglitz insurance market, where the sum of payoff functions is not upper semicontinuous. They show that the discontinuity is only on the diagonal element (i.e., on a set of points whose dimension is less than strategy space); thus, by taking the limit of mixed strategies of the finite game, the probability masses of the mixed Nash equilibrium strategy on the discontinuous points become zero. Thereby, they show the existence of a mixed Nash equilibrium.

In this paper, the problem is much more severe. As I have shown above, the discontinuity lies in off-diagonal elements: the neighborhood of $\hat{z}^f = ((A, g), (A, N.S.))$. It is not merely a point. The value of $g$ that creates this discontinuity can be anything below the optimal investment level of Firm 1 (e.g., $g < S(m, (A, N.S.))/5$ in case of $\alpha = 1/2$). Hence, the technique of Dasgupta and Maskin (1986b) cannot be applied.

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61 On the diagonal elements, the sum of the two functions is an upper hemicontinuous correspondence, because both are trying to become monopolists or free riders, and thus, production functions show no drastic change at
Reny (1999) is a latest work that specifies a sufficient condition for existence of a Nash equilibrium in a game with a discontinuous (single-valued) payoff function. The condition is called \textit{better-reply secure}: if for every nonequilibrium strategy $z^f*$ and every payoff vector limit $\pi^{f*}$ resulting from strategies approaching $z^f*$, some player $j$ has a strategy yielding a payoff strictly above $\pi^{f*}_j$ even if the others deviate slightly from $z^f*$.

The \textit{better-reply security} is a generalized condition of Dasgupta and Maskin’s diagonal discontinuity and weak lower semicontinuity conditions.

Again, consider a similar example to $\hat{z}^f$, but assume Firm 1 offers $(A,s^*)$ that maximizes its profit by free riding, given Firm 2’s offer $(A,N.S.)$, and also assume that Firm 1 can sell out its bond to all the households. For example, in case of $\alpha = 1/2$, $s^* = s(m_i, (A,N.S.)) / 5$. Each household buys one issue of Firm 1’s bonds and then buys bonds from Firm 2, up to $4s^*$. Note that a set of strategies described here is an example of nonequilibrium strategies: It is optimal for Firm 1 but not optimal for Firm 2.

Suppose Firm 2 slightly changes the coupon rate and offers, $(A + \epsilon, N.S.)$. Firm 1, the free rider, needs to match the coupon rate to $A + \epsilon$, because a coupon rate higher than Firm 2 would cost more, and a coupon rate lower than Firm 2 would attract no demand. The best strategy that Firm 1 can take, then, is to refine the offered amount at the new rate $A + \epsilon$.

I will prove, below, that Firm 1 cannot earn a higher profit with this slight change in Firm 2’s coupon rate than what it earns under the original strategies. This violates the \textit{better reply} condition. More formally, Firm 1’s maximized profit is proven to be monotonically changing in Firm 2’s coupon rate or unchanged at all. Therefore, Firm 1 cannot be strictly better off if Firm 2 changes its bond offer with a slightly different coupon rate that lowers (or does not affect) the residual demand for Firm 1’s bonds.

Consider that Firm 2 offers a contract with a slightly higher coupon rate, $(A + \epsilon, N.S.)$, and Firm 1 matches its coupon rate to $A + \epsilon$ with reoptimized amount. The aggregate savings will change to $\tilde{S} = S(m, (A + \epsilon, N.S.))$. I also denote the capital level of Firm 1 and Firm 2 at this rate as $\tilde{k}_1$ and $\tilde{k}_2$, respectively. The resource constraint is

$$\tilde{S} = \tilde{k}_1 + \tilde{k}_2.$$  \hspace{1cm} (A4)
Given Firm 2’s capital and knowing that it sells out all the bonds, Firm 1 chooses the offer amount \( \tilde{k}_1 \) to maximize the profit or equate the private marginal product of capital to the new coupon rate,
\[
\alpha A \tilde{k}^{1-\alpha} \tilde{k}_1^{\alpha-1} = A + \epsilon. \tag{A5}
\]
That is,
\[
\tilde{k}_1 = \alpha \frac{1}{\xi} \tilde{k}_2, \tag{A6}
\]
where
\[
\xi = \frac{A}{A + \epsilon}. \tag{A7}
\]
Using the resource constraint (A4), Firm 2’s capital is determined as the aggregate savings net of Firm 1’s capital:
\[
\tilde{k}_2 = \frac{1}{1 + \alpha \frac{1}{1-\alpha} \xi^{1-\alpha}} \tilde{S}. \tag{A8}
\]
Then, Firm 1’s reoptimized capital level at rate \( A + \epsilon \) can be expressed as
\[
\tilde{k}_1 = \frac{\alpha \frac{1}{1-\alpha} \xi^{1-\alpha}}{1 + \alpha \frac{1}{1-\alpha} \xi^{1-\alpha}} \tilde{S}. \tag{A9}
\]
Firm 1’s reoptimized profit is now expressed as
\[
\tilde{\pi}_1 = A \tilde{k}^{1-\alpha} \tilde{k}_1^{\alpha} - (A + \epsilon) \tilde{k}_1
= A \frac{\alpha \frac{1}{1-\alpha} \xi^{1-\alpha}}{1 + \alpha \frac{1}{1-\alpha} \xi^{1-\alpha}} \tilde{S} - A \frac{\alpha \frac{1}{1-\alpha} \xi^{1-\alpha}}{\xi} \frac{\alpha \frac{1}{1-\alpha} \xi^{1-\alpha}}{1 + \alpha \frac{1}{1-\alpha} \xi^{1-\alpha}} \tilde{S}
= A \frac{(1 - \alpha) \alpha \frac{1}{1-\alpha} \xi^{1-\alpha}}{1 + \alpha \frac{1}{1-\alpha} \xi^{1-\alpha}} \tilde{S}. \tag{A10}
\]
The derivative with respect to \( \epsilon \) is
\[
\frac{\partial \tilde{\pi}_1}{\partial \epsilon} = A \frac{\partial \tilde{S}}{\partial \epsilon} \frac{1}{1 + \alpha \frac{1}{1-\alpha} \xi^{1-\alpha}} \left( (1 - \alpha) \alpha \frac{1}{1-\alpha} \xi^{1-\alpha} \right) \frac{\frac{\alpha \frac{1}{1-\alpha} \xi^{1-\alpha}}{1 - \alpha} \frac{\partial \xi}{\partial \epsilon}}{\frac{\alpha \frac{1}{1-\alpha} \xi^{1-\alpha}}{1 - \alpha} \frac{\partial \xi}{\partial \epsilon}}
- A \frac{\partial \tilde{S}}{\partial \epsilon} \frac{(1 - \alpha) \alpha \frac{1}{1-\alpha} \xi^{1-\alpha}}{1 + \alpha \frac{1}{1-\alpha} \xi^{1-\alpha}} \left( 2 \right) \frac{\alpha \frac{1}{1-\alpha} \xi^{1-\alpha} \frac{\partial \xi}{\partial \epsilon}}{1 - \alpha} \frac{\partial \xi}{\partial \epsilon}
+ A \frac{(1 - \alpha) \alpha \frac{1}{1-\alpha} \xi^{1-\alpha}}{1 + \alpha \frac{1}{1-\alpha} \xi^{1-\alpha}} \frac{\partial \tilde{S}}{\partial \epsilon}. \tag{A11}
\]
The coefficients on $\partial \xi / \partial \epsilon$ can be combined to

$$A\tilde{S} \frac{1}{1 + \alpha \frac{\alpha^{1 - \alpha}}{1 - \alpha^{1 - \alpha}}} \left(1 - \alpha\right) \frac{1}{1 - \alpha} \frac{\xi^{\frac{\alpha}{1 - \alpha}}}{1 - \alpha^{\frac{1 - \alpha}{1 - \alpha}}} \left\{ \frac{1}{\xi} - \frac{\alpha^{\frac{\alpha}{1 - \alpha}} \xi^{\frac{\alpha}{1 - \alpha}}}{1 + \alpha^{\frac{1 - \alpha}{1 - \alpha}}} \right\}$$

\[= A\tilde{S} \alpha \frac{\alpha^{\frac{\alpha}{1 - \alpha}} \xi^{\frac{\alpha}{1 - \alpha}}}{1 + \alpha^{\frac{1 - \alpha}{1 - \alpha}}} \left\{ \frac{1}{\xi} - \frac{\alpha^{\frac{\alpha}{1 - \alpha}} \xi^{\frac{\alpha}{1 - \alpha}}}{1 + \alpha^{\frac{1 - \alpha}{1 - \alpha}}} \right\}. \tag{A12} \]

Now, temporarily use

$$\Lambda = \frac{\alpha^{\frac{\alpha}{1 - \alpha}} \xi^{\frac{\alpha}{1 - \alpha}}}{1 + \alpha^{\frac{1 - \alpha}{1 - \alpha}}}. \tag{A13}$$

Then, the derivative of Firm 1’s optimized profit regarding Firm 2’s coupon rate can be expressed as

$$\frac{\partial \tilde{\pi}_1}{\partial \epsilon} = A\tilde{S} \left\{ (1 - \alpha) \Lambda \frac{\partial \tilde{S}}{\partial \epsilon} + \alpha \Lambda \left( \frac{1}{\xi} - \Lambda \right) \frac{\partial \xi}{\partial \epsilon} \right\}. \tag{A14}$$

By multiplying both sides by $\epsilon / \tilde{\pi}_1$ and letting $\eta$ denote the elasticity of the aggregate Walrasian savings function, $\eta \equiv (\partial \tilde{S} / \partial \epsilon)(\epsilon / \tilde{S})$, (A14) can be expressed as

$$\frac{\partial \tilde{\pi}_1}{\partial \epsilon} \frac{\epsilon}{\tilde{\pi}_1} = A\tilde{S} \frac{\epsilon}{\tilde{\pi}_1} \left\{ (1 - \alpha) \Lambda \eta + \alpha \Lambda \left( \frac{1}{\xi} - \Lambda \right) \frac{\partial \xi}{\partial \epsilon} \right\}. \tag{A15}$$

The left-hand side is the elasticity of the maximized profit function of Firm 1 with respect to the coupon rate.

The second term in the brace of (A15) becomes zero in the limit of $\epsilon \to 0$. To see this, note that all elements other than $\epsilon$ are bounded:

$$\lim_{\epsilon \to 0} \xi = 1, \quad \lim_{\epsilon \to 0} \Lambda = \frac{\alpha^{\frac{\alpha}{1 - \alpha}}}{1 + \alpha^{\frac{1 - \alpha}{1 - \alpha}}}, \quad \text{and} \quad \lim_{\epsilon \to 0} \frac{\partial \xi}{\partial \epsilon} = -\frac{1}{A}. \tag{A16}$$

Therefore, in the limit, (A15) becomes

$$\lim_{\epsilon \to 0} \left( \frac{\partial \tilde{\pi}_1}{\partial \epsilon} \frac{\epsilon}{\tilde{\pi}_1} \right) = A\tilde{S} (1 - \alpha) \frac{\alpha^{\frac{\alpha}{1 - \alpha}}}{1 + \alpha^{\frac{1 - \alpha}{1 - \alpha}}} \eta.$$

Firm 1’s maximized profit increases with Firm 2’s coupon rate, if this elasticity of Firm 1’s maximized profit, with respect to Firm 2’s coupon rate, is positive. The sign is equal to the sign of $\eta$, the elasticity of the aggregate Walrasian savings function with respect to the interest rate. This depends on relative size of substitution and income effects, ultimately determined by the curvature of the utility function (e.g., intertemporal elasticity of substitution), given the initial wealth and allocation of firm ownership.

In summary, when there is a slight increase in Firm 2’s coupon rate, Firm 1’s maximized
profit is strictly increasing for $\eta > 0$, strictly decreasing for $\eta < 0$, or unchanged for $\eta = 0$. However, each of three cases violates the better-reply security condition. To see this, suppose $\eta > 0$. For a set of nonequilibrium strategies $(A, s^*)$ and $(A, N.S.)$, when Firm 2 slightly lowers the coupon rate and offers $(A - \epsilon, N.S.)$, Firm 1 has no strategy that yields strictly higher profit than the profit under the original set of strategies. Similarly, if $\eta < 0$, Firm 1 has no strategy that yields strictly higher profit when Firm 2 slightly raises the coupon rate. If $\eta = 0$, Firm 1 has no strategy that yields strictly higher profit when Firm 2 slightly raises or lowers the coupon rate.