HOUSEHOLD SAVING BEHAVIOR AND SOCIAL SECURITY PRIVATIZATION

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Standard model of saving:

- all households earn common rate of return

Data:

- heterogeneity in financial choices and outcomes

Questions:

- what are the causes and consequences of heterogeneity in financial decisions?
- how does this heterogeneity impact our thinking about social security reform?
Desired model features:

- financial outcomes are endogenous
- optimizing intermediaries also respond to policy
- bad financial outcomes reflect lack of information
- realistic wealth distribution
- general equilibrium (social security reform is big!)

Abstracting from:

- risky assets and diversification
- long-lived matches between households and intermediaries
Implementation:

- life-cycle Bewley-Huggett-Aiyagari model
- search to learn about investment opportunities
- offer distribution is endogenous

heterogeneity in wealth, income, age
⇒ heterogeneity in search effort
⇒ heterogeneity in financial outcomes
Predictions:

- increasing search effort as retirement approaches
- 60% asset market participation rate
- wealth associated with higher returns
- additional skewness in distribution of wealth
**Application: Social Security Privatization**

Two perspectives:

1. Privatization will undermine the social safety net
2. Fixed costs of portfolio management lead social security to be more distortionary

Experiment:

- Simulate a partial privatization of social security and compute welfare impact of policy announcement
- Compare to same experiment without search friction
Application: Social Security Privatization

Results:
In response to privatization:

▶ households increase search effort
▶ asset market participation rate rises
▶ degree of competition in asset market rises slightly
▶ larger response in capital, labor supply

Welfare analysis:

▶ privatization produces welfare loss without search friction (0.9% of consumption)
▶ search friction leads to additional welfare loss (1.1% of consumption)
THE MODEL

Population:
Continuum of households that follow a life cycle.

Life cycle:
- born at age 21
- age-dependent mortality risk
- retire at 66
- die for sure at 110

At death, replaced by children that inherit assets.

Population growth: household of age \( i \) replaced by \( (1 + \gamma)^i \) children.
Preferences

Parents are altruistic.

Dynastic preferences:

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left[ c_t^\chi (1 - n_t - s_t)^{1-\chi} \right]^{1-\rho} \]
Endowments

Time endowment used for:
- labor (while young)
- leisure
- search

Labor productivity depends on
- age \((i)\)
- discrete idiosyncratic shock

Household’s exogenous state: \(\epsilon \equiv (i, \tilde{\epsilon})\) follows a Markov chain.

Labor productivity given by \(y(\epsilon)\).
**Technology**

Production:
- \( k^\alpha \ell^{1-\alpha} + (1 - \delta)k \)
- only operated by firms

Storage:
- zero net return
- can be used by households
Firms rent capital and labor services from households (mass $\mu_t$ of firms).

Labor market:

- Walrasian market
- cleared by wage $w_t$
- labor input chosen after capital is in place
- $\Rightarrow$ common marginal product of capital: $A_t$
Firms pay a fixed cost $\psi$ to post a return (full commitment).

Households choose search effort $s \geq 0$.

Effort $s$ generates a stochastic number of meetings

$$\text{Prob}[j \text{ meetings}|s] = \frac{(\theta(s))^j e^{-\theta(s)}}{j!},$$

with

$$\theta(s) = \theta_1 \times \log (1 + \theta_2 \times s).$$
Market for capital services

With each meeting, household gets a draw from $F_t(r)$.

Households choose the highest return of those encountered.

Matches last for one period.

Comments:

Risky assets are not modeled explicitly.

Savings decision is made before search outcome realized.
Social insurance

Social security:

- benefit depends on final labor productivity: $B(\epsilon)$
- funded through tax on labor income: $\tau_t^y$
**Household’s problem**

\[ V_t(a, \epsilon) = \max_{c,a^+,s,n} \left\{ u(c, 1 - n - s) + \beta E_t \left[ V_{t+1}(a', \epsilon') \right] \right\} \]

such that

\[
\begin{align*}
c &= a + (1 - \tau_t - \tau_t^y) ny(\epsilon) w_t + B_t(\epsilon) - a^+ \\
a^+ &\geq 0 \\
a' &= [1 + r (1 - \tau_{t+1})] \times a^+ \\
r &\sim G_t(\cdot; s, \epsilon)
\end{align*}
\]
Firm’s problem

\[
\max_r \sum_{\epsilon} \int \frac{A - 1 - r}{A} h(a, \epsilon) \left\{ \mu^{-1} \theta(s(a, \epsilon)) e^{\theta(s(a, \epsilon))[F(r) - 1]} \right\} \Phi(da, \epsilon)
\]

Household decision rules:
\[h(a, \epsilon)\] for \(a^+\)
\[s(a, \epsilon)\] for \(s\)

Households are weighted by their savings.
FIRM EQUILIBRIUM

Partial equilibrium concept.

DEFINITION
A firm equilibrium is an offer distribution $F(\cdot)$ and a profit level $\pi^*$ such that

1. for all $r$ in the support of $F(\cdot)$, $\pi(r) = \pi^*$
2. for all $r$ outside the support of $F(\cdot)$, $\pi(r) \leq \pi^*$.

In equilibrium with search, $F(r)$ will

- have support $[0, \bar{r}]$
- be continuous
Firm equilibrium

- **eq’m offer distribution**
  
  - $F(r)

- **match probability**
  
  - $1 - \frac{r}{A-1}$

- **profit margin**
  
  - $1 - \frac{r}{A-1}$

- **expected profit**
  
  - $\pi(r)$
Calibration

5 year model period.

Population growth and survival probabilities from US data.
**Labor productivity**

\[ \log y (\epsilon) = \log \bar{y}_i + \zeta_t \]

Life-cycle component from PSID.

\( \zeta_t \) follows discretized AR(1)

Intergenerational transmission of productivity:
- correlation of parent’s and child’s earnings
Social security

Benefits chosen to broadly match US system.

9.3% payroll tax to clear social security budget.
Calibration: search process

\( \theta_1 \) and \( \theta_2 \) chosen to match

- average time spent on search
- median intermediation spread \( A - (1 + r) \)

Data:

- average time spent on finances in ATUS (3 minutes per day)
- median fee on S&P 500 index funds
Search mechanics

$\theta(s) = \text{expected number of offers}$

Graph showing the relationship between search effort and the expected number of offers.
Search mechanics

probability of asset market participation

search effort ($\times 10^{-3}$)
Search mechanics

expected return (annualized)

marginal product of capital

highest offered return

search effort ($\times 10^{-3}$)
SEARCH DECISION RULES

The graph shows the relationship between assets and search effort for high and low labor productivity. The search effort is expressed as search effort ($\times 10^{-3}$). The graph indicates that as the number of assets increases, the search effort increases for both high and low labor productivity. However, the search effort curve for low labor productivity is steeper than that for high labor productivity, indicating a higher search effort for low labor productivity at any given number of assets.
Life-cycle profile of search effort

average time spent on financial management

age
minutes per day

20 30 40 50 60 70 80

0 1 2 3 4 5 6

data
model
Asset market participation

60% of model households participate.

Data:
- 50% hold equities (including indirect holdings)
- 65% hold some financial assets other than checking or savings accounts.
Life-cycle profile of participation

![Graph showing asset market participation rate vs age]

- Fraction of households
- Age
- Asset market participation rate
- Model
- Data

The graph illustrates the fraction of households involved in asset market participation across different ages. The model curve peaks around age 60, while the data shows a peak around age 50, followed by a decline.
The distribution of offered returns

distribution of offers over returns

fraction of offers

return

distribution of S&P 500 index funds over fees

distribution of S&P 500 index funds over fees

fraction of funds

fee
The distribution of households over returns

distribution of households over returns

return

fraction of households

return

0 0.01 0.02 0.03 0.04 0.05
THE DISTRIBUTION OF ASSETS OVER RETURNS

distribution of assets over returns

distribution of assets over fees

fee

classification of assets over returns

fraction of assets
The distribution of wealth

Wealthy are more likely to participate in asset market...

...and earn higher returns (on average) conditional on participation.

Marginal effect of one st. dev. increase in wealth:

► participation rate increases by 17 percentage points,
► return increases by 13 basis points.
# The distribution of wealth

<table>
<thead>
<tr>
<th></th>
<th>Gini</th>
<th>Top Groups</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td>1%</td>
</tr>
<tr>
<td><strong>Data</strong></td>
<td>0.80</td>
<td>34.7</td>
</tr>
<tr>
<td><strong>Without Search Friction</strong></td>
<td>0.74</td>
<td>14.6</td>
</tr>
<tr>
<td><strong>With Search Friction</strong></td>
<td>0.78</td>
<td>16.5</td>
</tr>
</tbody>
</table>

Data notes: Calculations by Budria Rodriquez et al. (2002) using 1998 SCF.
**Impact of the offer distribution**

**Experiment:** solve the model with an exogenous offer distribution.

Use distribution of fees on S&P 500 index funds.

Recalibrate to same moments.

Savings, search effort, participation, and realized return distribution are not much different.
Social security
Privatizing social security

A partial privatization experiment:
  - benefits are gradually cut to 50% of initial levels
  - payroll tax reduced immediately by more than 50%
  - transition costs are funded by consumption tax
  - income taxes fall as economy expands

Privatization vs. phase-out.

Perform the experiment with and without the search friction.
Welfare impact of policy announcement

Welfare metric: change in average expected utility.

Average expected utility falls by 1.1% of consumption.

falls by 0.9% without the search friction.

Search friction reduces welfare across the state space.
## Long-run response of aggregates

<table>
<thead>
<tr>
<th></th>
<th>with search friction</th>
<th>without search friction</th>
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</thead>
<tbody>
<tr>
<td>output</td>
<td>4.94%</td>
<td>4.81%</td>
</tr>
<tr>
<td>capital</td>
<td>10.47%</td>
<td>10.26%</td>
</tr>
<tr>
<td>labor supply</td>
<td>3.60%</td>
<td>3.49%</td>
</tr>
<tr>
<td>average search effort</td>
<td>19.3%</td>
<td>–</td>
</tr>
<tr>
<td>asset market participation</td>
<td>11.7%</td>
<td>–</td>
</tr>
<tr>
<td>change in median fee</td>
<td>-4 BPS</td>
<td>–</td>
</tr>
</tbody>
</table>
CHANGE IN SEARCH EFFORT

average search effort

initial steady state

new steady state

average search effort

minutes per day

initial steady state

new steady state
CHANGE IN PARTICIPATION

asset market participation rate

fraction of households

age

initial steady state

new steady state

20 30 40 50 60 70 80
0.4 0.5 0.6 0.7 0.8 0.9 1

fraction of households

age
## Long-run welfare impact

<table>
<thead>
<tr>
<th></th>
<th>with search friction</th>
<th>without search friction</th>
</tr>
</thead>
<tbody>
<tr>
<td>average welfare at birth</td>
<td>6.08%</td>
<td>6.69%</td>
</tr>
</tbody>
</table>
CONCLUSIONS

Saving behavior:

Modeled endogenous heterogeneity in rates of return across households.

Plausible explanation of:

- limited asset market participation
- wealthier and higher income households more likely to participate/earn higher returns
- some skewness in distribution of wealth

Social security privatization:

Search friction leads to additional welfare loss of 0.2% of consumption.
Appendix
How does social security affect welfare?

\[ U(s) = (a - \tau) \left[ A - (A - 1) e^{-s\theta} (1 + s\theta) \right] - ws + b(\tau) \]

Search externality:

\[ \frac{\partial}{\partial s} U(s) \bigg|_{s=s^*} = w \]

Marginal benefit of social security:

\[ \frac{d}{d\tau} U(s) \bigg|_{s=s^*} = \frac{\partial}{\partial \tau} U(s) + \frac{\partial}{\partial s} U(s) \frac{\partial s}{\partial \tau} \]

\[ = \left[ \frac{\partial b}{\partial \tau} - A \right] + \left[ A - E(1 + r|s) \right] + w \frac{\partial s}{\partial \tau} \]
<table>
<thead>
<tr>
<th></th>
<th>total impact</th>
<th>across ages</th>
<th>across asset levels (percentile)</th>
<th>across income levels (shock)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>21 - 25</td>
<td>41 - 50</td>
<td>56 - 65</td>
</tr>
<tr>
<td>full model</td>
<td>-1.19%</td>
<td>0.89%</td>
<td>-3.31%</td>
<td>-2.52%</td>
</tr>
<tr>
<td>benchmark model</td>
<td>-0.99%</td>
<td>1.09%</td>
<td>-3.02%</td>
<td>-2.41%</td>
</tr>
</tbody>
</table>
SEARCH EXTERNALITY

The diagram illustrates the concept of symmetric cooperative equilibrium in search externality. The axes represent $S_i$ and $S_{-i}$, and the curve shows the relationship between the search efforts of an individual and others. The 45° line indicates the no search equilibrium, whereas the symmetric cooperative equilibrium is marked at point C. Points A, B, and C represent different states of search efforts, with A and B indicating different scenarios involving search efforts, and C representing the cooperative equilibrium.
Robustness to firm heterogeneity

Firm equilibrium relies heavily on homogeneity and exact indifference across support of $F(\cdot)$.

What if firms are heterogeneous?

Suppose $A_j \in [\underline{A}, \overline{A}]$.

As $\{A, \overline{A}\} \to A$, distribution of offered returns converges point-wise to $F(\cdot)$.

Take away: the equilibrium would not be too different with a small degree of heterogeneity among firms.
Spill-overs from search effort

The graph shows the relationship between search effort (s) and the wedge in planner's FOC. The graph indicates two types of externality:

- **Positive externality**: An upward arrow points to the right above the zero line, indicating a positive impact on the wedge as search effort increases.
- **Negative externality**: A downward arrow points to the left below the zero line, indicating a negative impact on the wedge as search effort increases.
Spill-overs from search effort

Search effort (s)

Wedge in planner’s FOC

Negative externality

Positive externality

Negative near s = 0

Positive around s = 1/Θ

Goes to zero for large s

Diagram showing the relationship between search effort (s) and the wedge in planner’s FOC, highlighting positive and negative externality effects.
Explanation of match probability

\[ \text{Prob}\{\text{match}|a,\epsilon,r\} \]

\[ \sum_{j=0}^{\infty} \text{Prob}\{j \text{ meetings and match}|a,\epsilon,r\} \]

\[ \sum_{j=0}^{\infty} \text{Prob}\{j \text{ meetings}|a,\epsilon,r\} \times \text{Prob}\{\text{match}|j \text{ meetings},a,\epsilon,r\} \]

\[ \sum_{j=0}^{\infty} \mu^{-1} j \left[\theta_s(a,\epsilon)\right]^j e^{-\theta_s(a,\epsilon)} \frac{j!}{j!} \times [F'(r)]^{j-1} \]

\[ \mu^{-1} \theta_s e^{\theta_s[F(r)-1]} \]
Solution algorithm for steady state

Typical BHA model:

- For a given capital-labor ratio, calculate \( w \) and \( A \).

- Solve household problem and simulate.

- Check capital-labor ratio.
Solution algorithm for steady state

Iterative procedure with two nested loops:

- For a given capital-labor ratio calculate \( w \) and \( A \)
  - Given an \( F(\cdot) \), derive \( G(r; s) \) and solve the consumer’s problem.
  - Simulate using the policy rules, \( F(\cdot) \) and \( \Gamma \) to find the stationary distribution \( \Phi \).
  - Using \( \Phi \) and policy rules, the firm profit equation gives a new \( F(\cdot) \)

- Check capital-labor ratio.

Solving for \( F(\cdot) \):
\[
\frac{\partial \pi}{\partial r} = 0 \text{ on support of } F(\cdot). \text{ ODE w/ initial condition } F(0) = 0.
\]
Multiple equilibria?
Modify definition from standard BHA model to include:

- given $A$, $h$, $s$, and $\Phi$, \( \{F(\cdot), \pi^*\} \) is a firm equilibrium
- firms have no incentive to enter or exit: $\pi^* = \psi$
- given $G(r, s)$ and $w$, $V$ solves household’s problem
- $G(r, s)$ is consistent with $F(r)$
- $K = \sum_\epsilon \int h(a, \epsilon) [1 - q(0; a, \epsilon)] \Phi(da, \epsilon) - \mu\psi$
- $\Phi(A, \epsilon') = \Phi(A, \epsilon)$
- $\Phi(A, \epsilon') = \sum_\epsilon \varGamma(\epsilon' ; \epsilon) \int \int (1+r) \times h(a, \epsilon) \in A G(dr; s(a, \epsilon), \epsilon) \Phi(da, \epsilon)$
**Other parameters:**

<table>
<thead>
<tr>
<th>symbol</th>
<th>description</th>
<th>value</th>
<th>target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>discount factor (yearly)</td>
<td>0.97</td>
<td>capital-output ratio</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Cobb-Douglas parameter</td>
<td>0.36</td>
<td>capital share</td>
</tr>
<tr>
<td>$\delta$</td>
<td>depreciation rate (yearly)</td>
<td>0.067</td>
<td>investment rate</td>
</tr>
<tr>
<td>$\rho$</td>
<td>risk aversion</td>
<td>2</td>
<td>–</td>
</tr>
<tr>
<td>$\chi$</td>
<td>labor supply parameter</td>
<td>0.373</td>
<td>average hours</td>
</tr>
<tr>
<td>$\psi$</td>
<td>fixed cost</td>
<td>1</td>
<td>normalization</td>
</tr>
<tr>
<td>$\tau$</td>
<td>income tax rate</td>
<td>0.194</td>
<td>avg. marg. tax rate</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>consumption tax rate</td>
<td>0.0</td>
<td>–</td>
</tr>
<tr>
<td>$\tau_y$</td>
<td>payroll tax</td>
<td>0.093</td>
<td>s.s. budg. balance</td>
</tr>
</tbody>
</table>
Fixed offer distribution: median asset profile

Median assets at beginning of period

- Exogenous offer distribution
- Baseline

Normalized assets vs. age
Fixed offer distribution: average search effort profile

![Graph showing average search effort across age, with lines for exog. offer distrib. and baseline.](image-url)
**Fixed offer distribution: financial participation rate**

The graph shows the asset market participation rate as a function of age, with two lines indicating different scenarios: exogenous offer distribution and baseline. The x-axis represents age, ranging from 20 to 80, and the y-axis represents the fraction of households, ranging from 0 to 1.
The distribution of households over returns

Baseline

Exogenous distribution
When household searches \( s \), the highest return is drawn from

\[
G(r; s) \equiv \sum_{j=0}^{\infty} \frac{(\theta(s))^j e^{-\theta s}}{j!} [F(r)]^j
\]

\[
= e^{\theta(s)[F(r)-1]}.
\]