Inferring Income Risk from Economic Choices: An Indirect Inference Approach

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A Stochastic Process for Labor Income

\( y_t^i : \log \) labor earnings of household \( i \) at age \( t \).

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y_t^i = \left[ a_0 + a_1 t + a_2 t^2 + a_3 \text{Educ} + \ldots \right]
\]

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where

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where \( z^i_t = \rho z^i_{t-1} + \eta^i_t \), and \( \eta^i_t, \epsilon^i_t \sim iid \)
Three Questions about Labor Income Risk

1. How persistent and large are income shocks? i.e., what is $\rho$ and $\sigma^2_\eta$?

2. Do individuals differ systematically in their income growth rates? i.e., is $\sigma^2_\beta \gg 0$?

3. If indeed $\sigma^2_\beta \gg 0$, how much do individuals know about their $\beta^i$ at different points in their life-cycle?

Main conclusion:

Typical calibrations of incomplete markets models substantially overstate uninsurable income risk.
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Existing Evidence from Labor Income Data

1 HIP Process ("Heterogenous Income Profiles"): 
   - Early studies estimated the full model and found:
     \[ \sigma^2_{\beta} \gg 0 \quad \text{and} \quad 0.5 \leq \rho \leq 0.8 \]
     

2 RIP Process ("Restricted Income Profiles"): 
   - MaCurdy (1982) suggested a test for \( \beta_i \equiv 0 \) and could not reject it.
   - Then he and the following literature:
     - Imposed \( \beta_i \equiv 0 \) and estimated \( 0.95 \leq \rho \leq 1.0 \).
     

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Studies the joint dynamics of consumption and labor income to learn more about labor income risk.

Two Difficulties

● First, GMM requires strong assumptions.
  ▶ “Indirect inference” circumvents many of these difficulties.

● Second, long US panel on consumption does not exist.
  ▶ We construct a panel of imputed consumption (1968-1992) by combining CEX and PSID.
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A Life Cycle Model

- A standard life-cycle model of consumption-savings choice (CRRA utility, borrowing constraints, retirement system).

- Information Structure. Recall: \( y_t^i = \alpha^i + \beta^i t + z_t^i + \epsilon_t^i \)
  - Bayesian learning about \((\beta^i, z_t^i)\) observing \(y_t^i\) and \(\epsilon_t^i\).
  - Cast learning as a Kalman filtering problem.

- Express the prior standard deviation as: \(\sigma_{\beta,0} = \lambda \sigma_\beta\).
  - If \(\lambda = 0\) \(\rightarrow\) \(\sigma_{\beta,0} = 0\) (No prior uncertainty).
  - If \(\lambda = 1\) \(\rightarrow\) \(\sigma_{\beta,0} = \sigma_\beta\) (Full prior uncertainty).

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Identification

Simplifying assumptions for intuition:

- (i) quadratic utility,
- (ii) no borrowing constraints,
- (iii) $\varepsilon_t \equiv 0$, and
- (iv) $y_t$: level of income.

1. Optimal consumption choice:

   $$\text{HIP: } \Delta C_t = \Pi_t \times \left( y_i^t - \left( \alpha_i^t + \beta_{t-1}^i t + \rho \hat{z}_{t-1}^i \right) \right)$$

2. If $\sigma_{\beta}^2 \equiv 0$, learning disappears $\rightarrow$ we get (certainty equivalent) permanent income model:

   $$\text{RIP: } \Delta C_t = \Psi_t \times \eta_t$$
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Optimal consumption choice:

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1. Identification from Consumption Changes

Guvenen and Smith (2010)

Inferring Income Risk from Choices
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HIP: \( \Delta C_1 < 0, \Delta C_2 > 0 \)

\( \eta = \xi^2 > 0 \)

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2. Identification from Consumption Levels

![Graph showing consumption levels and labor earnings over age for Agent 1 and Agent 2.]

Guvenen and Smith (2010)
2. Identification from Consumption Levels

RIP:
\[ C^1_{t=3} = Y_{t=3} = C^2_{t=3} \]

Forecast Income Paths under RIP
2. Identification from Consumption Levels

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Guvenen and Smith (2010) Inferring Income Risk from Choices
Indirect Inference

- We estimate the structural model using “indirect inference.”

- This approach provides a way to choose which moments to match.

- Imposes far few restrictions on the structural model than GMM.

- Monte Carlo analysis shows that the indirect inference method works very well.
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Why Indirect Inference?

The standard method since Hall and Mishkin (1982) is to derive structural equations explicitly and estimate them.

- For example, with (i) quadratic utility, (ii) no borrowing constraint, (iii) no retirement, and (iv) \( Y_t = z_t + \varepsilon_t \), and \( z_t = z_{t-1} + \eta_t \), we have:

\[
\Delta C_t = \eta_t + \psi_t \varepsilon_t \quad \psi_t \sim 0
\]

- Persistence can be measured by \( p \equiv \sigma_\eta/(\sigma_\eta + \sigma_\varepsilon) \)

\[
\Delta C_t = \Delta Y_t \quad \text{if } p = 1 \quad \text{(permanent shocks)}
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Response of consumption growth to income growth reveals persistence of income shocks.
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- $\Rightarrow$ Response of consumption growth to income growth reveals persistence of income shocks.
An Example

An Example: Binding Constraints

Guvenen and Smith (2010)
A Feasible Auxiliary Model:

\[
\Delta C_t = \Pi_t \times \left( Y_t^i - \left( \alpha + \hat{\beta}^i_{t-1} t + \rho \hat{z}^i_{t-1} \right) \right)
\]

This regression is not feasible, so approximate with

\[
c_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + a_3 y_{t+1} + a_4 y_{t+2}
+ a_5 \bar{y}_{1,t-3} + a_6 \bar{y}_{t+3,T} + a_7 \Delta \bar{y}_{1,t-3} + a_8 \Delta \bar{y}_{t+3,T}
+ a_9 c_{t-1} + a_{10} c_{t-2} + a_{11} c_{t+1} + a_{12} c_{t+2} + \text{error}
\]

where \( c_t \equiv \log (C_t) \).

Add a second regression where \( y_t \) is the dependent variable. Use the same income regressors above.

Guvenen and Smith (2010) Inferring Income Risk from Choices
A Feasible Auxiliary Model:

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Monte Carlo Results

<table>
<thead>
<tr>
<th>True Value</th>
<th>Estim. mean</th>
<th>Estim. std</th>
</tr>
</thead>
</table>
| **Income Processes Parameters:**
| $\sigma_\alpha$ | 0.284 | 0.279 | 0.025 |
| $\sigma_\beta$ | 1.852 | 1.815 | 0.176 |
| $\text{corr}_{\alpha\beta}$ | –0.162 | –0.146 | 0.148 |
| $\rho$ | 0.754 | 0.758 | 0.025 |
| $\sigma_\eta$ | 0.196 | 0.196 | 0.005 |
| $\sigma_\epsilon$ | 0.004 | 0.030 | 0.023 |
| **Economic Model Parameters:**
| $\lambda$ | 0.345 | 0.348 | 0.084 |
| $\delta$ | 0.950 | 0.950 | 0.002 |
| $\psi$ | 0.874 | 0.869 | 0.096 |
| **Measurement Errors:**
| $\sigma_y$ | 0.147 | 0.142 | 0.007 |
| $\sigma_c$ | 0.356 | 0.356 | 0.002 |
| $\sigma_{c0}$ | 0.428 | 0.422 | 0.009 |
## Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std Error</th>
<th>Description</th>
</tr>
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<tbody>
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<td>$\rho$</td>
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<td>0.025</td>
<td>persistence</td>
</tr>
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<td>$\sigma_\eta$</td>
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<td>0.005</td>
<td>std. dev. of perm. shock</td>
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<tr>
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Quantifying Life Cycle Income Risk

Within-Cohort Variance of Log Income

\[
\text{Var}(\log(\text{Income}))
\]

US Data

Guvenen and Smith (2010)
Quantifying Life Cycle Income Risk

Guvenen and Smith (2010)  
Inferring Income Risk from Choices  
23. Oktober 2010  
18 / 18
Conclusion 1: Less than 1/3 of cross-sectional income dispersion at retirement represents risk—the rest is known heterogeneity.
Conclusion 2: Existing estimates in the literature overstate labor income risk by a factor of 3 to 5.