Finance and Misallocation:
Evidence from Plant-Level Data∗

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Abstract

We study a model of establishment dynamics in which entrepreneurs face a financing constraint. We ask: does the model, when parameterized to match salient features of plant-level data, predict large aggregate TFP losses from misallocation? Our answer is: No. In our model efficient establishments quickly accumulate internal funds and grow out of their borrowing constraint. The model thus predicts TFP losses from misallocation, even in an economy with no external finance, that are at most 5-7%. This is not an impossibility result. We present parameterizations of the model in which finance frictions cause substantially larger TFP losses. Such parameterizations are, however, at odds with important features of plant-level data: the variability and persistence of plant-level output, as well as differences in the return to capital and output growth rates across young and old plants.

Keywords: Productivity. Misallocation. Finance Frictions

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1. Introduction

Differences in GDP per capita across countries are large and mostly accounted for by differences in total factor productivity. A key question in economic development is thus: What accounts for the large cross-country differences in total factor productivity? One hypothesis that has received much attention recently\(^1\) is that of establishment-level misallocation. According to this hypothesis, poor countries are poor not only because individual establishments are less efficient, but also because factors of production are not optimally allocated among establishments.

We study, in this paper, the role of credit constraints in generating misallocation and therefore aggregate TFP losses. Our motivation stems from the observation that finance and TFP are strongly correlated in the data. Figure 1 illustrates this by showing a scatterplot of TFP versus a measure of how developed financial markets are: the ratio of external finance (private credit and stock market capitalization) to GDP for a sample of countries for 1996\(^2\).

The question we ask is, To what extent does this correlation reflect the effect finance frictions have on resource allocation among productive units? Do finance frictions cause large TFP losses by hindering the process of reallocation?

The mechanism we study is the following. Consider an environment in which agents differ in their ability to operate a technology that is subject to decreasing returns. Entrepreneurial ability evolves over time. In such an environment efficiency dictates that the marginal product of capital and labor be equal across entrepreneurs. Entrepreneurs that become more productive over time should accumulate a greater share of the economy’s stock of capital and labor. If entrepreneurs are financially constrained, however, they may be unable to acquire the optimal amount of capital and labor. Finance frictions can thus distort allocations among existing productive units. Moreover, they can also distort entry into entrepreneurship if productive individuals are poor and unable to borrow.

Our goal in this paper is to measure the strength of this mechanism. We study, through the lens of a model of establishment dynamics with financing frictions, plant-level data from manufacturing plants in Korea and Colombia. We choose these two countries as they provide relatively high quality micro-level data, but also because the two differ substantially in the level of financial development. Korea is a country with relatively well-functioning credit

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\(^1\)Restuccia-Rogerson (2008), Hsieh and Klenow (2008)

\(^2\)We use the TFP data from Caselli (2005) and the data on finance from Beck et. al. (2000).
markets with an external finance to GDP ratio equal to 120%, while Colombia has relatively poor credit markets and an external finance to GDP ratio of around 30%.

We require that our model accounts for a number of salient features of the micro data: the size distribution of establishments, the variability and persistence of output at the plant level, as well as the difference in returns to capital and output growth rates for young and old plants. We show that all these features of the data help pin down parameter values that are critical in determining the strength of the mechanism.

We find that, when we parameterize our model to account for these plant-level facts, this mechanism is fairly weak. The model predicts that the TFP gains from moving from an environment with no external finance to the level of external finance observed in the US are about 5-7%. Although this is not a trivial amount, such a TFP gap is small relative to the dispersion in TFP in the cross-country data. As the solid line in Figure 1 shows, the average TFP gap in the data between countries with no external finance and the US is about 60%.

This is not an impossibility result: we present alternative parametrizations of the model that can generate much larger TFP losses, as large as 30%. We show however that such parameterizations miss important features of the plant-level data, most notably the dispersion in output growth rates across establishments, as well as the dispersion in returns to capital across young and old plants. We thus argue that plant-level data imposes tight bounds on the model’s predictions regarding the strength of the relationship between finance and TFP.

The economy we study is a model of industry dynamics as in Hopenhayn (1992). A continuum of entrepreneurs differ in the efficiency with which they can operate a plant. Efficiency fluctuates over time, thus giving rise to micro-level dynamics and the need for external credit to finance expansions. We assume, given the evidence in Moskowitz and Vissing-Jorgensen (2002), that entrepreneurial risk is not diversified and that dividends from the establishment are the only source of income for entrepreneurs. Plant owners can save using a one-period risk-free security, but the amount they can borrow is subject to a collateral constraint, as in Evans and Jovanovic (1989).

We study two versions of the model. In the first version there is no entry and exit. We show that in this environment the key parameter that determines the relationship between finance and TFP is the standard deviation of shocks to an entrepreneur’s productivity. The larger the shocks are, the greater the need for external borrowing to finance expansions, and
hence the greater the losses from the borrowing constraint. Since variation in productivity is the sole source of variation in output in the model, we pin down the size of productivity shocks by requiring that the model matches moments of the distribution of output growth rates among establishments. We find that there is simply too little time-series variability in output in the data (in both Korea and Colombia) for productivity shocks to distort allocations much. Intuitively, absent large productivity shocks, efficient but poor entrepreneurs face large returns to investment and therefore accumulate internal funds quickly. This process of internal accumulation, absent large time-series variation in productivity, implies that efficient agents do not stay constrained for long and the economy achieves a fairly efficient allocation of resources.

Clearly, the process of internal accumulation, which prevents productive agents from staying constrained too long, is key to our findings of small TFP losses. In the model, a key parameter that determines the extent to which entrepreneurs can accumulate internal funds is the rate of time-preference: impatient entrepreneurs grow much more slowly out of their borrowing constraint. We show in the paper that the rate of time-preference, which governs the entrepreneur’s ability to substitute internal for external funds, has important implications for the model’s predictions about the relationship between the capital-output and external debt-to-output ratio. The more impatient entrepreneurs are, the less their ability to accumulate internal finance, and thus the more sensitive is an economy’s capital stock to the amount of external finance. Our model, we show, does an excellent job at reproducing the relationship between the capital-to-output and debt-to-output ratio in the data, suggesting that entrepreneurs can indeed accumulate internal funds.

We then turn to the second version of the model. Here we allow entry and exit into entrepreneurship by introducing a occupational choice: agents must decide whether to work or become entrepreneurs. Productive agents that have sufficient funds to operate at a large enough scale enter entrepreneurship, while the rest work. In addition, we assume a constant death hazard each period. Agents that die lose all their assets and are replaced by young agents that receive an endowment that is potentially correlated with their productivity. The constant death hazard is necessary in order to allow the model to account for the fact that some very large establishments shut down in the data. Moreover, without exogenous exit, establishment exit and entry plays little role since only marginal entrepreneurs with low productivity switch occupations and these account for too small a share of aggregate output.
for this margin to be quantitatively important.

In this second environment another key parameter that determines the size of aggregate TFP losses is the extent to which a newly born agent’s endowment is correlated with its ability as an entrepreneur. If all newborn agents have a small endowment, then productive agents join entrepreneurship but are initially very constrained. In such an environment TFP losses from misallocation are quite large. We show, however, that the predictions of such a model are at odds with the characteristics of young and old plants in the data. In particular, young plants grow much faster in the model than in the data and have much greater returns to capital (as measured by the average product of capital) than old plants. Hence, the model generates large TFP losses for the wrong reasons, by implying that young plants are much more severely borrowing constrained than they are in the data.

These counterfactual predictions can be remedied by allowing a newly born agent’s initial endowment to be correlated with its productivity (for example due to seed funding by venture capitalists of the high-potential entrepreneurs). When we choose the correlation between the initial endowment and productivity to match the differences in output growth rates and rates of return to capital among young and old plants in the data, we find once again fairly small TFP losses from misallocation. Importantly, most of these losses arise due to misallocation of capital across existing plants, not due to distortions in the occupational choice.

Our paper is related to a number of recent studies that quantitatively examine the impact of financing frictions on the level of economic development. These studies have generally found an important causal role for finance in accounting for TFP. For example, Jeong and Townsend (2006) attribute 70% of Thailand’s TFP growth from the 70s to the 90s to an improvement of the financial sector. Amaral and Quintin (2010), Buera, Kaboski and Shin (2009), Moll (2010), Greenwood, Sanchez and Wang (2010) also provide careful quantitative assessments of the effect of finance on misallocation. The TFP losses that these studies report are staggering: TFP would double if one were to increase access to external finance in poor countries to levels similar to those in developed countries like US.

Our contribution, relative to the existing work, is to discipline the quantitative analysis using a richer set of cross-sectional and time-series observations from establishment-level data. We find that a model that replicates the volatility of plant-level growth rates, as well as the difference in growth rates and returns to capital for young and old establishments, predicts
much smaller TFP losses than previous studies have found. We also show, in the context of our model, that ignoring these features of the data can lead one to find much greater TFP losses from misallocation. Our model does predict, as existing studies have found, that finance frictions have an important effect on the level of economic development, as measured by output. This effect arises, however, because of the effect of finance frictions on the economy’s stock of capital, rather than productivity.

Our paper is also related to a wider literature that has proposed a number of alternative theories of misallocation. In addition to credit frictions, this literature has also studied the role of distortionary government policies, frictions that distort factor mobility, as well as lack of insurance against the risk associated with entrepreneurial activity\textsuperscript{3}.

We proceed as follows. Section 2 presents the model without exit and entry. Section 3 discusses the data and the salient plant-level facts. Section 4 conducts the quantitative analysis. We then allow for entry and exit in Section 5.

2. Model

The economy is populated by a continuum of entrepreneurs, each of whom has access to a technology that produces output using inputs of capital and labor. Production is subject to decreasing returns to scale. All entrepreneurs produce a homogenous good and operate in a perfectly competitive environment. Because our focus is on aggregate TFP losses in the ergodic steady-state of a small open economy with no aggregate uncertainty, we conduct the analysis of this section in a partial equilibrium setup. The general equilibrium extension is relevant and pursued in Section 5 when we study the model with an occupational choice.

A. Environment

Let $i$ index an individual entrepreneur. Such an entrepreneur has an objective given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_{it}^{1-\gamma}}{1-\gamma}$$

where $C_{it}$ is consumption. The entrepreneur has access to a production technology given by:

$$Y_{it} = F(L_{it}, K_{it}) = A_{it}^{1-\eta} (L_{it}^{\alpha} K_{it}^{1-\alpha})^{\eta}$$

where $Y_{it}$ is output, $L_{it}$ is the amount of labor it hires, $K_{it}$ is the capital stock and $A_{it}$ is the entrepreneur’s productivity. The parameter $\eta \in (0, 1)$ is the span-of-control parameter and governs the degree of returns to scale$^4$. The share of labor in production is governed by $\alpha$. We assume that the log of productivity, $a = \log(A)$, follows a continuous-state Markov process with transition density

$$\Pr(a_{it+1} = a'|a_{it} = a) = \pi(a'|a)$$

We next describe the assumptions we make regarding the financial side of the model. We follow Buera, Kaboski and Shin (2010) and assume a setup similar to that in Evans and Jovanovic (1989) in which all debt is intra-period and agents cannot borrow intertemporally in order to smooth consumption$^5$.

Let $B_{it}$ denote an entrepreneur’s assets (financial and physical) at the end of period $t-1$, expressed in units of output, the numeraire. These assets are deposited with a financial intermediary and pay a risk-free interest rate $r$. At the beginning of period $t$ the entrepreneur must hire workers and install new capital. There are no capital adjustment costs: capital can thus be transformed into output and vice versa at a constant rate, normalized to 1. The key assumption we make is that factor payments must be made at the beginning of period $t$, before production takes place. Letting $W$ denote the wage rate, the entrepreneur must spend a total of $WL_{it} + K_{it}$ at the beginning of period $t$. We assume that the entrepreneur finances this expenditure by borrowing from financial intermediaries, also at an interest rate $r$. The amount the entrepreneur can borrow is limited, however, by a collateral constraint:

$$WL_{it} + K_{it} \leq \lambda B_{it}$$

$^4$Clearly, this formulation can be alternatively interpreted as arising from an environment in which monopolistically competitive firms face a constant elasticity demand function. Under this alternative interpretation $Y$ represents revenue and $\eta = 1 - 1/\theta$ where $\theta$ is the demand elasticity.

$^5$We redid our analysis in a model with intertemporal borrowing, in a Kiyotaki and Moore (1997) environment, and have found very similar results to those we report here.
where, recall, $B_{it}$ is the amount of funds the entrepreneur has deposited with the bank and $\lambda$ is a parameter that governs the strength of the borrowing constraint. On one hand, if $\lambda = 1$, the entrepreneur cannot borrow externally. On the other hand, if $\lambda = \infty$, the entrepreneur faces no within-period borrowing constraints. We refer to this economy as the frictionless economy. Finally, we define debt as

$$D_{it} = W L_{it} + K_{it} - B_{it}.$$ 

At the end of period $t$ production takes place and the entrepreneur receives $Y_{it} + (1 - \delta) K_{it}$, the output and the undepreciated portion of its capital stock. The entrepreneur then decides how much to consume, $C_{it}$, and how much to save, $B_{it+1} \geq 0$, subject to its budget constraint:

$$C_{it} + B_{it+1} = Y_{it} + (1 - \delta) K_{it} + (1 + r) [B_{it} - W L_{it} - K_{it}]$$

The budget constraint says the amount available for consumption and saving is equal to output and undepreciated capital, net of the debt payments of the entrepreneur\(^6\).

In this economy, if the entrepreneur is sufficiently patient, she quickly accumulates assets in order to avoid the borrowing constraint. To allow finance frictions to play a role, we must preclude entrepreneurs from accumulating assets. We do so here by assuming that the rate of time preference, $\beta$, is low relative to the rate at which agents can save, $r$. In particular, we assume $\beta (1 + r) < 1$.

**B. Recursive Formulation and Decision Rules**

We next discuss the decision rules in this environment. Since all debt is intratemporal, the entrepreneur’s problem of how much capital and labor to hire is static. We can thus first solve the entrepreneur’s profit maximization problem and then its consumption-savings decision. The profit maximization problem reduces to:

$$\Pi (B_{it}, A_{it}) = \max_{K_{it}, L_{it}} A_{it}^{1-\eta} \left( L_{it}^\alpha K_{it}^{1-\alpha} \right)^\eta - (1 + r) W L_{it} - (r + \delta) K_{it}$$

\(^6\)Since there are no capital adjustment costs here, there is no distinction between renting and owning capital. Our setup thus admits both interpretations.
subject to
\[ WL_{it} + K_{it} \leq \lambda B_{it} \]

We note that this problem is homogenous in \( A_{it} \). Let \( l_{it} = \frac{L_{it}}{A_{it}}, k_{it} = \frac{K_{it}}{A_{it}}, b_{it} = \frac{B_{it}}{A_{it}} \) denote labor, capital and assets rescaled by the entrepreneur’s productivity. We can thus write, dropping the \( it \) subscript:

\[
\pi (b) = \max_{k,l} \left( l^\alpha k^{1-\alpha} \right)^\eta - (1 + r) WL - (r + \delta) k \text{ s.t. } WL + k \leq \lambda b
\]

Clearly, the solution to this problem involves equating the marginal product of capital and labor to their user cost:

\[
f_l (l,k) = F_L (L,K) = [1 + \tilde{r} (b)] W
\]
\[
f_k (l,k) = F_K (L,K) = \tilde{r} (b) + \delta
\]

where

\[
\tilde{r} (b) = r + \mu (b).
\]

Here \( \mu (b) \) is the multiplier on the borrowing constraint and \( \tilde{r} (b) \) is the entrepreneur’s shadow cost of funds.

Notice that dispersion in \( \tilde{r} (b) \) is the only source of aggregate productivity losses in this economy. Productivity losses arise due to wedges in the marginal product of capital and labor, of the type examined by Hsieh and Klenow (2009) and Restuccia and Rogerson (2008), which here arise endogenously. To the extent to which financing frictions induce dispersion in the entrepreneur’s shadow cost of funds, \( \tilde{r} (b) \), the marginal products of capital and labor are not equalized across entrepreneurs. For the model to produce efficiency losses, two conditions must therefore be satisfied. First, the model must generate dispersion in \( b \), the amount of assets an entrepreneur has relative to its productivity. Second, at least some of the entrepreneurs must be borrowing constrained, so that dispersion in \( b \) translates into dispersion in the multiplier on the borrowing constraint, \( \mu (b) \).

We show next that there is a powerful force in this model that reduces the amount of dispersion in rescaled assets, \( b \). To see this, we next turn to the entrepreneur’s consumption-
savings decision. We can write the entrepreneur’s dynamic program as:

\[ V(b, a) = \max_{b' \geq 0} \frac{c^{1-\gamma}}{1-\gamma} + \beta \int \exp (a' - a)^{1-\gamma} V\left( \frac{b'}{\exp(a' - a)}, a' \right) \pi(a'|a) \, da' \]  

(1)

where

\[ c = (1 + r) b + \pi(b) - b'. \]

The optimal savings decision satisfies:

\[ c^{-\gamma} = \beta \int (1 + r + \mu') \exp \left( \frac{-\gamma}{1-\eta} (a' - a) \right) c^{-\gamma} \pi(a'|a) \, da' \]

where, recall, \( \mu \) is the multiplier on the within-period borrowing constraint. Notice that in this rescaled formulation of the problem, the entrepreneur’s productivity, \( a \), enters only through its effect on the conditional density \( \pi(a'|a) \).

Figure 2 summarizes the optimal decision rules\(^7\) by showing the relationship between asset holdings, \( b \), and the shadow cost of funds, \( \tilde{r}(b) \), as well as savings, \( b'(b) \). We contrast the decision rules in our economy with those in an economy with no borrowing constraints, i.e., in which \( \lambda = \infty \). Clearly, the greater the agent’s assets are, the less is its reliance on external funds and the lower the shadow cost of funds. Sufficiently rich entrepreneurs have an shadow cost of funds equal to the risk-free rate, \( r \). In contrast, poor entrepreneurs face a high shadow cost of funds. This is shown in Panel A of Figure 2.

Panel B shows the entrepreneur’s savings decision: its savings, \( b' \), expressed relative to its initial asset holdings, \( b \). Rich entrepreneurs dissave, \( b'/b < 1 \), since \( \beta (1 + r) < 1 \) and entrepreneurs are impatient. Poor entrepreneurs, in contrast, accumulate assets, \( b'/b > 1 \), since their returns to savings, \( r + \mu \), are greater.

To build some intuition for what determines the ergodic distribution of \( b \) in this economy, assume for a moment that entrepreneurial productivity is constant over time, but not necessarily across entrepreneurs. Clearly, as evident in Figure 2, absent changes in productivity the distribution of assets would collapse over time to a mass point at which the Euler

\(^7\)We use projection methods and Gaussian quadrature to compute the solution to the entrepreneur’s problem.
equation reduces to:

\[ 1 = \beta [1 + r + \mu (b)] \]

This is the case regardless of the underlying amount of dispersion in productivity across entrepreneurs. Even though all entrepreneurs are borrowing constrained at this point (since \( \beta (1 + r) < 1 \)), they face the same shadow cost of funds and the allocation of capital and labor across establishment is efficient.\(^8\)

It follows that changes in productivity are necessary in order for finance frictions to induce dispersion in the shadow cost of funds. Shocks to productivity, as the Bellman equation (1) shows, act like shocks to any given entrepreneur’s rescaled asset holdings. In particular, a positive productivity shock lowers the entrepreneur’s rescaled assets, causing an increase in the internal cost of funds and therefore the marginal product of capital and labor. Hence, finance frictions can generate large dispersion in the marginal product of capital and labor – and therefore large aggregate productivity losses, only if changes in productivity are sufficiently large. We ask whether this is indeed the case in our empirical analysis below.

To further illustrate the workings of the model, Figure 3 shows the impulse response to a temporary increase in the entrepreneur’s productivity, \( a \). The evolution of the entrepreneur’s productivity is shown in Panel A of the figure: we assume a mean-reverting AR(1) process in this particular example. The increase in productivity erodes the entrepreneur’s rescaled assets, thus raising its shadow cost of funds, as shown in Panel B of the figure. Since the borrowing constraint binds, the entrepreneur cannot raise its stock of capital (and labor) sufficiently. Hence, as Panel C illustrates, its stock of capital increases gradually. Since the entrepreneur is constrained and faces a high rate of return, it finds optimal to accumulate assets (Panel D). This allows it to eventually grow out of its borrowing constraint.

To summarize, increases in an entrepreneur’s productivity give rise to a tightening of the borrowing constraint. Finance frictions act here much like physical adjustment costs and slow down the response of capital and labor to productivity shocks. The difference between finance frictions and physical adjustment costs is that the former imply an asymmetric response to positive and negative productivity shocks. Entrepreneurs can respond more easily to negative productivity shocks since these make it optimal to sell capital and labor and thus relax the collateral constraint.

\(^8\)See Banerjee and Moll (2009) who formalize this idea.
The fact that more productive entrepreneurs in our model are more severely con-
strained may seem counter-intuitive, especially in light of the results of Kiyotaki and Moore
(1997). The difference between our setup and that of Kiyotaki and Moore is that they study
the response of the model economy to an aggregate productivity shock. An aggregate pro-
ductivity shock in their model increases the price of capital and thus relaxes the borrowing
constraint. This latter effect is absent here because we consider idiosyncratic productivity
shocks that have no effect on prices.

C. TFP losses from misallocation

We next describe how we compute total factor productivity and the losses from mis-
allocation in our model economy. Consider the problem of allocating the aggregate stock of
capital $K = \int K_i \, di$ and labor, $L = \int L_i \, di$ in this economy so as to maximize total output:

$$\max_{K_i, L_i} Y = \int A_i^{1-\eta} \left( L_i^\alpha K_i^{1-\alpha} \right) \eta \, di$$

s.t. $K = \int K_i \, di$ and $L = \int L_i \, di$

Clearly, the solution to this problem requires that the returns to factors are equal
across entrepreneurs and that the allocations of capital and labor satisfy:

$$L_i = \frac{A_i}{\int A_i} L \text{ and } K_i = \frac{A_i}{\int A_i} K.$$  

Then aggregate output is equal to

$$Y = TFP \left( L^\alpha K^{1-\alpha} \right)^\eta$$

and TFP satisfies:

$$TFP = \frac{Y}{\left( L^\alpha K^{1-\alpha} \right)^\eta} = \left( \int_0^1 A_i \right)^{1-\eta}.$$  

Consider next the economy with borrowing frictions. Now the optimality conditions
are:

\[ f_l(l, k) = w(1 + \tilde{r}) \text{ and } f_k(l, k) = \tilde{r} + \delta \]

and the marginal products of capital and labor are no longer equal across entrepreneurs. The labor and capital allocations satisfy

\[ L_i = \frac{\omega^l_i A_i}{\int A_i di} L \text{ and } K_i = \frac{\omega^k_i A_i}{\int A_i di} K. \]

where the wedges \( \omega^l_i \) and \( \omega^k_i \) are decreasing in the cost of internal funds, \( \tilde{r}_i \):

\[
\omega^l_i \sim (\tilde{r}_i + \delta)^{-\frac{(1-\alpha)\eta}{1-\eta}} (1 + \tilde{r}_i)^{-\frac{1-(1-\alpha)\eta}{1-\eta}},
\]

\[
\omega^k_i \sim (\tilde{r}_i + \delta)^{-\frac{1-\alpha\eta}{1-\eta}} (1 + \tilde{r}_i)^{-\frac{\alpha\eta}{1-\eta}}.
\]

We can then still write

\[ Y = TFP \left( L^\alpha K^{1-\alpha} \right)^\eta, \]

where the productivity level is a function of the distribution of wedges and entrepreneurial productivity \( A_i \):

\[ TFP = \frac{\int_0^1 \omega_i^A A_i di}{(\int A_i di)^\eta} \]

where \( \omega_i = (\omega^l_i)^\alpha (\omega^k_i)^{1-\alpha} \).

We thus have that TFP decreases in the dispersion of wedges, \( \omega_i \), as long as \( \eta < 1 \). Moreover, TFP decreases if the covariance between wedges (labor and capital shares) and productivity is reduced. Generating large TFP losses from misallocation thus requires that the model generates sufficient dispersion in the marginal product of capital and labor and/or that productive entrepreneurs cannot accumulate the optimal amount of capital and labor.

**3. Data**

We next discuss the source of the plant-level data we use and the strategy we employ to pin down values for the key parameters of the model. We then study the model’s implications for the relationship between aggregate productivity and the economy’s debt-to-GDP ratio.
A. Data Description

We use data for two countries, the more financially developed South Korea, as well as the less financially developed Colombia, a country in which the external finance to GDP ratio is one-fourth of that in South Korea. We next describe each of the two datasets in part.

Korea

The data we use are from the Korean Annual Manufacturing Survey, which is collected by the Korean National Statistical Office. The survey is conducted every year from 1991 to 1998, except for the year of the Industrial Census (1993) for which we supplement the data using the Census data (which covers all establishments). The survey covers all manufacturing plants with five or more workers.

The survey reports information about each plant’s total revenue, number of employees, total wage bill, payments for intermediate inputs (materials), as well as energy use. The survey also reports the book value of a plant’s capital stock, as well as purchases, retirement/sales, and depreciation for land, buildings, machinery and equipment. This information allows us to construct a measure of plant-level capital using a perpetual inventory method, using the reported book value of capital to initialize each series and augmenting each year’s series to include purchases net of depreciation and retirements. We follow earlier work and focus on buildings, machinery and equipment as our measure of capital stock. Finally, we augment each plant’s stock of capital to include the amount it leases. We define labor expenditure as wage and benefit payments to workers. The intermediate inputs include raw materials, water, and fuel. All series are real.

We drop observations that are clearly an outcome of coding errors: observations with negative values for revenue, expenditure of labor and intermediate inputs, and book value of capital. Our sample consists of about 400,000 plant-year observations over an eight year period from 1991 to 1998. We focus on the 1991-1996 period, the years before the financial crisis.

We augment the data using information from the Bank of Korea Financial Statement Analysis on the aggregate debt-to-value added ratio in Korean manufacturing. The Financial Statement Analysis is a survey of all large firms as well as a stratified random sample of smaller firms. The aggregate debt-to-sales ratio of firms in this dataset is equal to 0.50,

\[9\] See e.g. Caballero et al. (1995).
implying a debt-to-GDP ratio equal to 1.2 (this number is very close to that reported in Beck et. al (2000) for this period).

**Colombia**

The data are from the Colombian Industrial Survey and covers the years 1986 to 1991. The Survey collects data on all establishments with more than 10 workers. The survey provides information on the book value, purchases, sales, and depreciation of capital. This allows us to construct measures of capital stock in a similar fashion as for the Korean data described above. We measure labor expenditure as wage and benefit payments. Intermediate inputs include energy, raw materials, and fuels. All series are real.

After excluding observations that are an obvious outcome of coding error using the same criteria as in Korean data, we are left with about 40,000 plant-year observations for 1986 to 1991. Finally, Beck et. al. (2000) report that the external debt to GDP ratio in Colombia is equal to 0.30 in this period, thus much smaller than that in Korea.

**B. Establishment-level facts**

We next describe several features of the plant-level data. These are not unique to the particular countries we study: many of these have been documented in earlier work\(^\text{10}\). We present these features here in order to guide our quantitative analysis below. Since the economy we study assumes no entry and exit, we focus now on a balanced panel of plants in both countries that are continuously in sample throughout the years for which we have data available. Roughly 32,000 plants are fit this criterion in Korea and 5,000 plants in Colombia. We later show that the facts we document below are very similar when we study the entire (unbalanced) panel of plants.

Since the process for productivity is what primarily determines the size of aggregate productivity losses, we focus on features of the data that allow us to pin down this process. Although we do not directly observe an individual’s plant productivity, we note that, since productivity is the sole source of variation in output in our model, we can identify its process by requiring the model to account for the distribution of output and its growth rate across plant in our data. This is the approach taken in most quantitative studies of establishment dynamics.

The measure of output in the data that most closely corresponds to that in our model is

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\(^{10}\)See for example Rossi-Hansberg and Wright (2007).
value added, i.e., revenue net of expenditure on intermediate inputs, since we have abstracted from the latter in the theory. We thus report salient features of the data on value-added in the two manufacturing panels we study.\footnote{Our results are robust to introducing intermediate inputs as a factor of production and using data on revenue instead of value added.}

**Distribution of output growth rates**

Panel A of Table 1 shows that output growth rates are very dispersed across plants in the data. The standard deviation of changes in the log of value added from one year to another\footnote{All the statistics we report are for the pooled sample of plants in all years, but most of the variation reflects cross-sectional dispersion, rather than variation across years.} is equal to 0.54 in Korea and 0.49 in Colombia. In addition to dispersion, we report a number of higher-order moments of the growth rate of output. First, notice that the distributions show excess kurtosis (fat tails): the kurtosis of growth rates is equal to 13 in Korea and 21 in Colombia. For comparison, the kurtosis of a Gaussian is equal to 3. Thus, a small number of plants experience very large increases or declines in their output. Another way to gauge the thickness of the tails (the kurtosis itself is very sensitive to outliers) is to compare the interquartile range of the distribution to the standard deviation. The former is, by definition, unaffected by the shape of the tails, while the standard deviation is. Notice in Table 1 that the interquartile range is smaller than the standard deviation in both datasets: 0.49 in Korea and 0.36 in Colombia. For comparison, the interquartile range of a Gaussian is about 1.3 times larger than its standard deviation.

**Persistence and scale dependent growth**

Panel B reports the correlation of a given plant’s log output with its lags at horizons of one, three and five years. Note that the first-order autocorrelation is equal to 0.93 for Korea and 0.96 for Colombia. Hence, despite a fair amount of variability in output growth rates, output across plants is fairly persistent. That these autocorrelations are below unity suggests that output tends to mean-revert, or that growth rates are negatively correlated with the size of the establishment. To see this, note that the first-order autocorrelations reported in Table 1 imply that the coefficient of a regression of output growth rates on lagged output:

$$y_{it} - y_{it-1} = (\rho - 1) y_{it-1} + \varepsilon_{it}$$
is equal to $\rho - 1 = -0.07$ (0.93 - 1) for Korea and $\rho = -0.04$ for Colombia. Larger plants therefore grow slower: doubling output tends to decrease a plant’s growth rate by about 7% in Korea and 4% in Colombia.

The table also reports higher-order autocorrelations. An important feature of the data is that these decay slowly with the horizon, so that output is much more persistent than suggested by the first-order autocorrelation. The autocorrelations at lags 3 and 5 are equal to 0.89 and 0.86 in Korea and 0.93 and 0.90 in Colombia. For comparison, an AR(1) process that decays geometrically with a serial correlation parameter equal to 0.93 (0.96) would imply much lower fifth-order autocorrelations: $0.93^5 = 0.69$ (0.96$^5 = 0.82$).

**Size distribution of establishments**

The final feature of the data we document is the size distribution of establishments. Panel C of Table 1 shows that output is heavily concentrated in a few large establishments: the largest 1% of establishments account for 57% (30%) of all manufacturing value added in Korea and Colombia, respectively. Similarly, the largest 20% of establishments account for 91% (88%) of all value added.

**Cross-country comparison**

The statistics reported above are fairly similar for the two countries. In both countries there is a lot of dispersion in plant-level growth rates, output is very persistent and concentrated in the largest plants. Perhaps the only noticeable difference is that output is slightly less concentrated and more persistent in Colombia than it is in Korea. One conjecture is that these differences reflect differences in the sampling criteria in these two datasets. While the Korean survey includes data on all plants with more than 5 workers, the Colombian data includes only plants with more than 10 workers. To see the role of these sampling differences, the last column of Table 1 reports statistics for a truncated sample of Korean plants with more than 10 workers, the criterion used for Colombia. Notice that the Korean numbers change very little when we eliminate the roughly 5000 plants that have fewer than 10 workers, thus suggesting that the differences between Colombian and Korean datasets do not reflect sampling differences.

4. Quantitative Analysis

Recall that our question is, what is the effect of finance frictions on aggregate productivity? To answer this question, we next study a quantitative version of the model parame-
terized to fit the salient plant-level facts described above. We next discuss the strategy we use to pin down the model’s parameters.

A. Parameterization

We group parameters into two categories. The first category includes parameters that are difficult to identify using our data. These include preference and production function parameters. We assign these values that are common in existing work and show below that our results are robust to perturbations of these parameter values. The second category includes parameters that determine the process for productivity at the micro-level, as well as the size of the financing frictions, which are the key determinants of the size of aggregate productivity losses in the model. We pin down values for these parameters by requiring that the model accounts for the salient features of the data discussed above.

Assigned Parameters

The period is one year. We set the intertemporal elasticity of substitution, governed by $\gamma$, equal to 1. We set the risk-free interest rate equal to 4% per year, $r = 0.04$. The discount factor, $\beta$, determines the extent to which entrepreneurs accumulate assets and hence their ability to grow out of the borrowing constraint. We follow Buera, Kaboski and Shin (2010) and use $\beta = 0.92$, implying that entrepreneurs are fairly impatient. We assign production function parameters that are standard in existing work: capital depreciates at a rate $\delta = 0.06$, the span-of-control parameter is equal to $\eta = 0.85$, as in Atkeson and Kehoe (2005) and a share of labor equal to $\alpha = 2/3$. The latter choice allows the model to match the expenditure share of labor in value added in our data.

Calibrated Parameters

The rest of the parameters are jointly pinned down by the requirement that the model accounts for the plant-level facts. We use an indirect inference approach to estimate these parameter values, by choosing parameter values that minimize the distance between a number of plant-level moments in the model and in the data.

Since we would like our model to simultaneously account for a number of features of the data, we assume a somewhat more complex process for entrepreneurial productivity. In

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13 Other values of $\eta$ used in recent work include 0.75 (Bloom 2009), 0.79 (Buera, Kaboski and Shin (2010), 0.82 (Bachmann, Caballero and Engel 2010) and 0.90 (Khan and Thomas 2008). See also the work of Basu and Fernald (1995, 1997).
particular, we assume that entrepreneurial productivity is the sum of two components:

\[ a_{it} = Z_i + \tilde{a}_{it} \]

where \( Z_i \) is a fixed (permanent) productivity component. We assume that \( \exp(Z_i) \) is distributed according to a Pareto with an upper bound \( H \) and a shape parameter, \( \mu \):

\[ \Pr[\exp(Z_i) \leq x] = \frac{1 - x^{-\mu}}{1 - H^{-\mu}}. \]

We think of \( Z_i \) as capturing time-invariant ‘entrepreneurial’ ability. The other component of productivity is an AR(1) process:

\[ \tilde{a}_{it} = \rho \tilde{a}_{i,t-1} + \varepsilon_{it}. \]

We illustrate below the role of these two components and show that both are important in allowing the model to account for the plant-level facts\(^\text{14}\).

Since we have documented that the distribution of output growth rates shows excess kurtosis, we allow for fat-tailed shocks to the variable productivity component. In particular, we assume that the shocks, \( \varepsilon_{it} \) are drawn from a mixture of Normals:

\[ \varepsilon_{it} \sim \begin{cases} N(0, \sigma_1^2) & \text{with prob. } 1 - \kappa \\ N(0, \sigma_2^2) & \text{with prob. } \kappa \end{cases} \]

where \( \sigma_2^2 > \sigma_1^2 \) and \( \kappa \) determines the probability with which shocks are drawn from the more dispersed distribution. Intuitively, fat-tailed shocks have the potential to amplify the size of aggregate TFP losses since they imply that a small fraction of firms experience very large increases in productivity, thus amplifying their need for external borrowing.

\(^\text{14}\)That \( Z_i \) is time-invariant is not crucial. We have obtained similar results by assuming \( Z_i \) follows a unit root process.
aggregate TFP (see e.g. Buera, Kaboski and Shin (2010) and Moll (2010)) is to calibrate the model to data from a relatively undistorted economy (usually the U.S.) and then trace out the effect of varying the collateral constraint, \( \lambda \), on aggregate productivity. We follow a similar strategy here. In particular, we pin down key parameters of the model by requiring that the model accounts for the establishment-level facts in the relatively more financially developed Korea. We then hold all other parameters constant and trace out the effect of \( \lambda \) on aggregate productivity and other implications of the model.

**Objective Function**

The moments we use are the 11 moments that characterize the plant-level facts in Table 1 for Korea. We have 8 parameters to calibrate: \( \theta = \{ \lambda, \rho, \sigma_{1,\varepsilon}, \sigma_{2,\varepsilon}, \kappa, \sigma_z, \mu, H \} \). To pin down these parameters, we use an indirect inference approach. Let \( \Gamma^d \) denote the 11 \( \times \) 1 vector of moments in the data. Let \( \Gamma (\theta) \) denote the vector of moments in the model. Let \( W \) denote the inverse of the variance-covariance matrix of the data moments, computed by bootstrapping repeated samples of the data with replacement. We pin down \( \theta \) by minimizing the following objective:

\[
\min_{\theta} \left[ \Gamma (\theta) - \Gamma^d \right]' W \left[ \Gamma (\theta) - \Gamma^d \right]
\]

(2)

Table 2 (column labeled Benchmark) presents the parameter values that minimize the objective (2)\(^{15}\). The collateral constraint, \( \lambda \), is equal to 2.58, implying a leverage ratio \( D/B \) equal to 1.58. The serial correlation of the persistent productivity component, \( \rho \), is equal to 0.74. The standard deviation of shocks is equal to \( \sigma_{1,\varepsilon} = 0.52 \) and \( \sigma_{2,\varepsilon} = 1.78 \) and firms draw from the more volatile distribution \( \kappa = 0.07 \) of the time. Finally, the permanent productivity component has a shape parameter \( \mu = 3.64 \) and an upper bound \( \log H = 9.02 \). Together, these parameters imply that the permanent component accounts for two-thirds of the cross-sectional variance of productivity. Intuitively, the permanent component must be sufficiently dispersed in order to allow the model to reconcile the concentration of output among the largest plants in the data together with the fairly strong mean reversion of output.

Table 3 reports how the model does at matching the moments in the data. The fit is very good, reflecting the rich process for productivity we have assumed. The model accounts well for the distribution of output growth rates, the pattern of serial correlation and the high

\(^{15}\)We have also computed standard errors for these parameters and can make them available upon request. These are extremely small, reflecting the sensitivity of the moments we chose to the parameters we calibrate, as well as the large number of observations in the data.
concentration of output among the largest plants. The root mean square error, computed as

\[
RMSE = \left( \frac{1}{11} \sum_{i=1}^{11} \left( \ln (\Gamma_i^d) - \ln \Gamma_i (\theta) \right)^2 \right)^{\frac{1}{2}}
\]

is equal to 4.4% and most of this is accounted for by a slight mismatch between the concentration statistics in the model and data\(^{\text{16}}\).

**Constrained model (no permanent component)**

We next ask: what is the role of the permanent productivity component? To answer this question we estimate a constrained version of the above model, by eliminating the permanent component, \(Z_i\). We re-calibrate this economy using the same strategy and same vector of moments as above. Table 2 reports the parameter values that best fit the data and Table 3 reports how the models do at matching the establishment-level facts.

Eliminating the permanent productivity component worsens the model’s fit considerably. Since there is too much scale dependence in growth, large plants do not stay large for long and the model misses both the size distribution of plants in the data, as well as the autocorrelation pattern. Absent a permanent productivity component, there is a trade-off between matching the dispersion of the growth rates on one hand, and the large concentration of plants in the data on the other hand. The root mean square error in this version of the model is much higher and equal to 23.5%.

**B. Model Predictions**

We next discuss the model’s predictions about the extent to which entrepreneurs are constrained and the size of aggregate productivity losses it generates.

**Size of financing frictions**

Recall that the shadow cost of funds (the effective interest rate at which an entrepreneur borrows) is equal to

\[\tilde{r}_{it} = r + \mu_{it}\]

\(^{16}\)The fit can be further improved by allowing for an additional, iid, component of productivity (noise). Adding this additional component, however, has no effect on our results, so we do not report the results of this experiments here. Details are available from the authors upon request.
where $r$ is the risk-free rate and $\mu_{it}$ is the multiplier on the borrowing constrained. Clearly, since $\bar{r}_{it}$ determines an entrepreneur’s user cost of capital and labor, dispersion in $\bar{r}_{it}$ is the sole source of aggregate TFP losses in this economy. We first report several measures of the extent to which $\bar{r}_{it}$ is dispersed in the ergodic steady-state of our economy. Note, in Panel A of Table 4 that 54% of entrepreneurs are financially constrained, and the median premium ($\bar{r}_{it} - r$) they face, if constrained, is equal to 0.025, thus a 62% premium over the risk-free rate. The interquartile range of the premium is 0.03, the 90th percentile of the premium is 0.07, while the 99th percentile is equal to 0.15.

As anticipated, productive entrepreneurs are more likely to be constrained, since they are the ones who need to increase their stock of capital and labor and do not have sufficient funds to do so. We illustrate this in Figure 4. We group entrepreneurs into percentile of the $a$ distribution and, for each percentile, compute the average shadow cost of funds across entrepreneurs in that category. Figure 4 shows that very unproductive entrepreneurs face a shadow cost of funds equal to 0.04, the risk-free rate, while the very productive entrepreneurs face a shadow cost of funds equal to 0.08, thus an 100% premium.

**TFP losses from misallocation**

Table 4 also reports the size of aggregate TFP losses in our Benchmark economy, computed as described above. These amount to 3.9%, a fairly large number relative to, say, those in Hopenhayn and Rogerson (1993), but small when compared to the 30% TFP gap between Korea and the US in 1996.

To understand how these losses vary with the degree of financial development, we next conduct a number of experiments in which we vary $\lambda$ and hold all other parameters constant. We study the model’s implications for several values of $\lambda : \lambda = \{50, 1.2, 1\}$, each chosen to match a debt-to-GDP ratio equal to $\{2.3, 0.3, 0\}$, as in the U.S., Colombia and in an economy with no external borrowing.

As Table 4 shows, changing the debt-to-GDP ratio has a noticeable impact on establishment-level statistics. While very few (4%) entrepreneurs are constrained in the ‘U.S.’ calibration, the majority (80%) are constrained in the ‘Colombia’ calibration, or in the economy with no external borrowing (86%). Moreover, financing frictions have an important role on the pattern of establishment-level dynamics: the standard deviation of changes in log-output is equal to 0.70 in the ‘U.S.’ calibration, and thus almost twice as high than in an economy with little external borrowing (0.37 in ‘Colombia’ and 0.35 in an economy with no external borrowing.
borrowing). Notice also that finance frictions are a source of fat-tails in the distribution of output growth rates. While kurtosis is fairly low (7.5) in the ‘U.S.’ calibration, it increases to almost 30 in an economy with little external finance. Finally, financing frictions make output more persistent: the serial correlation of output is equal to 0.92 in the ‘U.S.’ calibration and increases to 0.98 in an economy with little external finance. Intuitively, finance frictions impart persistence to output since they act much like an adjustment cost of capital: entrepreneurs react to good productivity shocks gradually by slowly accumulating internal funds and growing out of the borrowing constraint.

These predictions of the model regarding the effect of finance frictions on establishment-level dynamics are consistent with the pattern we have documented in the micro-data. Recall that Colombian plants experience less volatile output growth rates (the standard deviation is 0.49 vs. 0.54 in Korea, while the interquartile range is 0.36 vs. 0.49 in Korea), more persistent output (the serial correlation of output is 0.96 vs. 0.93 in Colombia) and a more fat-tailed distribution of output growth rates (the kurtosis of changes in output is equal to 13 in Korea and 21 in Colombia). In other words, the model fits the Colombian statistics fairly well, even though we have not explicitly calibrated it to do so.

Table 4 also reports the answer to our key question: what is the size of TFP losses induced by financing frictions? The table shows that, unlike establishment-level micro moments, aggregate TFP losses vary little across these different experiments. The “U.S.” economy implies TFP losses of 1%, the ”Colombia” economy predicts TFP losses of 5.4%, while an economy with no external borrowing has TFP losses equal to 6.9%. The model thus accounts for a small fraction (5% vs. 60%) of the aggregate TFP differences between financially developed countries like U.S. and countries with little external finance.

*Why are the TFP losses small?*

To see why the TFP losses are small here, recall that absent changes in productivity, the ergodic distribution of entrepreneurs would collapse to a mass point at $b_i = \bar{b}$, implying that all entrepreneurs would face the same shadow cost of funds. This would imply that the marginal product of capital and labor would be equalized across entrepreneurs and there would be no aggregate TFP losses from misallocation. Finance frictions can thus generate TFP losses only in an economy where the variable component of productivity, $\tilde{a}_{it}$, is sufficiently dispersed. It turns out, however, that this is not the case in the calibration of our model consistent with the micro data. To see this, we next compute the worst-case TFP
losses, i.e., those in an economy the capital and labor shares are independent of \( \tilde{a}_{it} \). Formally, we compare the TFP level in an efficient economy, in which \( L_{it}, K_{it} \) are proportional to an entrepreneur’s productivity, \( \exp (Z_i + \tilde{a}_{it}) \) to the TFP level in an economy in which labor and capital allocations are not measurable with respect to \( \tilde{a}_{it} \).

Table 4 shows that these losses are equal to 8.6% in our economy, thus suggesting that there is too little dispersion in the variable productivity component so that even the most extreme form of adjustment costs cannot distort aggregate TFP too much. Relative to this benchmark, finance frictions in our model are quite potent: they produce a substantial proportion of what the TFP losses would be absent any adjustment of factors of production to productivity shocks.

To summarize, the model predicts that the TFP losses from financing frictions are fairly small, even for economies with little external finance, thus much too small to account for the cross-country dispersion in TFP. The reason losses are small here is that productivity shocks must be fairly small in order for the model to account for the dispersion in output growth rates in the data. Small shocks imply that any form of adjustment costs (of which finance frictions are a special case) cannot distort allocations much in this environment.

C. Counterfactual experiments

We next conduct several counterfactual experiments in order to illustrate how ignoring key features of the plant-level data can lead to the conclusion that TFP losses from misallocation are, in fact, much larger. We report the results of these experiments in Table 5.

Consider the consequence of ignoring data on the dispersion of output growth rates. We eliminate the permanent productivity component and assume productivity follows a simple AR(1) process:

\[
a_{it} = \rho a_{i,t-1} + \varepsilon_{it},
\]

where \( \varepsilon_{it} \) is an iid, normal random variable with variance \( \sigma_{\varepsilon}^2 \). We choose the two parameters characterizing this process, \( \rho \) and \( \sigma_{\varepsilon}^2 \), to match a) the serial correlation of output in the data of 0.93, as well as b) statistics that characterize the degree of concentration of the size distribution of firms.

Panel I of Table 5 shows that this version of the model requires much more volatile productivity shocks in order to fit the size distribution of firms. Intuitively, since we have
eliminated the permanent component of productivity, there is too much mean-reversion, implying that rich establishments decline quickly and can never become too large unless shocks to productivity are large. Such shocks imply, however, that output growth rates are much more volatile than they are in the data: the standard deviation is 1.05 vs. 0.54 in the data, and the interquartile range is 1.22 vs. 0.49 in the data. Moreover, this version of the model predicts too little autocorrelation in an entrepreneur’s output at horizons longer than a year. Since the autocorrelation decays geometrically here, the 5th-order serial correlation is much smaller in the model (0.69) than it is in the data. Clearly, this counterfactual is greatly at odds with the micro data.

Notice also that the model now predicts substantially larger TFP losses from misallocation: 10.5% for Korea and 18.1% for Colombia, reflecting the large shocks to productivity that entrepreneurs cannot easily react to. These losses are approximately 3-4 times greater than in the Benchmark economy. However, the model generates these losses for the wrong reasons, by implying changes in output from one year to another that are much greater than in the data.

In a recent paper Moll (2009) has argued that plant-level productivity is much less persistent in the data than what we have estimated here. He estimates a serial correlation parameter ($\rho = 0.80$) that is much lower than what we have used in the experiment in Panel I ($\rho = 0.92$) and argues that the resulting TFP losses are much greater when the serial correlation of productivity is low.

The difference between our estimates of $\rho$ reflect differences in methodology. While Moll (2009) computes a Solow residual measure of plant productivity, we require that the model accounts for the serial correlation of output in the data. Given the difficulty of measuring productivity and the uncertainty regarding the value of $\rho$, we ask whether our results are indeed sensitive to the value of this parameter. We do so by assigning $\rho$ a value equal to 0.8 and recalibrating $\sigma_{1,\varepsilon}^2$ to allow the model to match the size distribution of establishments in the data.

Panel II of Table 5 reports the results of this experiment. Since now there is much faster mean reversion in productivity, even greater shocks to productivity ($\sigma_{1,\varepsilon} = 2.3$) are required to account for the size distribution of establishments in the data. Because shocks are more volatile, TFP losses are even greater now: 18% for Korea and almost 30% for Colombia. Once again, however, the model generates the large TFP losses for the wrong reasons, by implying
much too volatile plant-level dynamics: the standard deviation of output growth rates is now equal to 2.17, thus four times greater than in the data.

Notice also that a lower value of $\rho$, on its own, does not generate greater TFP losses, if one were to hold constant the standard deviation of shocks to productivity (rather than the unconditional variance of productivity). To see this, we set $\rho = 0.8$, but now keep the standard deviation of shocks equal to 1.19, the value in economy II. We report the results of this experiment in Panel III of Table 5. Notice that now TFP losses are, in fact, smaller than in the economy with more persistent shocks (6.6% vs. 10.5% earlier for, say, Korea). Thus, holding constant the standard deviation of shocks, more persistent productivity actually amplifies TFP losses. Intuitively, if the shocks are equally sized, a more persistent shock lasts for a larger number of periods, and since it takes a while for the entrepreneur to grow out of the borrowing constraint (see Figure 3), misallocation persists. In contrast, a more transitory shock reverts quicker to the mean, thus imply a more short-lived increase in the entrepreneur’s marginal product of capital.

We thus conclude that it is the standard deviation of changes in productivity, rather than the persistence of shocks, that accounts for the differences in TFP in the previous two counterfactual experiments. When we calibrated the standard deviation of changes in productivity to match the standard deviation of changes in output in the data, we found that the TFP losses from misallocation are small, regardless of the persistence of shocks we use.

D. Sensitivity

We next gauge the robustness of our results to our choices of several of the parameters that are difficult to identify using our plant-level data and that we have simply assigned values to. All of these economies are re-calibrated so that the model matches the same set of moments we have used earlier. Table 6 reports results in these experiments and the parameter values we have used.

Discount factor

Column II of Table 6 reports the results in an economy with a lower discount factor, $\beta = 0.85$. When entrepreneurs are more impatient they have less incentive to accumulate internal funds, and so the borrowing constraints are more severe. Indeed, the table shows that in this case the TFP losses from misallocation are almost twice larger. Finance frictions also induce larger differences in TFP across countries: the TFP gap between the US and
Colombia is now 6.6% (2.2% vs. 8.8%), thus a bit higher than earlier. Overall, however, our earlier conclusions do not change much: finance frictions generate a very small fraction of the TFP differences between rich and poor countries.

**Elasticity of substitution between capital and labor**

Column III of Table 6 reports the results from an economy in which capital and labor are more imperfectly substitutable. In particular, we assume that technology is

\[ Y_i = A_i^{1-\eta} \left[ \alpha L_i^{\theta \frac{\alpha+1}{\alpha}} + (1 - \alpha) K_i^{\theta \frac{\alpha+1}{\alpha}} \right]^{\theta \frac{\alpha}{\alpha+1}} \]

We set \( \theta \), the elasticity of substitution between capital and labor, equal to 0.5 and choose, \( \alpha \), the weight on labor in the production function, to ensure that the labor and capital share in value added are equal to those in the Cobb-Douglas experiment.

Table 6 that our results are very similar to those in the Cobb-Douglas experiments. Although now the labor allocations are more distorted than earlier by the borrowing constraints, there is too little time-series variation in productivity for adjustment frictions to distort aggregate TFP much.

**Span of control**

We next increase the span of control parameter, \( \eta \). Column IV of Table 6 shows that raising \( \eta \) to 0.95, a number at the upper range of values used in the literature, does not affect our results much: the TFP losses in Korea are equal to 5.6% (compared to 3.9% earlier), and the TFP gap between the US and Colombia is equal to 5% (6.9% vs. 2.9%).

The reason these numbers is similar to what we had earlier is that we have re-calibrated the process for productivity to match the same standard deviation of shocks. Holding constant the volatility of \( A^{1-\eta} \), a higher \( \eta \) would raise the TFP losses from misallocation since it would make it optimal for the more productive plants to accumulate a greater share of the economy’s stock of capital and labor. A greater \( \eta \), however, would also increase the volatility of output. Hence, when we re-calibrate the economy by lowering the volatility of shocks to \( A^{1-\eta} \), similar TFP losses obtain.
E. Cross-country evidence on $K/Y$

Our result that TFP losses from misallocation are small is driven by the process of internal accumulation. Productive entrepreneurs are able to accumulate internal funds and grow out of their borrowing constraint. In the model, the ability of entrepreneurs to accumulate internal funds is governed by the rate of time-preference, $\beta$. The lower is $\beta$ relative to the risk-free rate, the lower the incentive to accumulate internal funds and grow out of the borrowing constraint. Alternatively, wedges between the rate at which entrepreneurs can save and borrow also reduce the entrepreneur’s ability to save.

We next ask: does our model overestimate the ability of entrepreneurs to accumulate internal funds? To answer this question we study the model’s predictions regarding the relationship between the capital to output ration, $K/Y$, and the external finance to GDP ratio, and compare it to the data.

To see why this relationship is informative about the extent to which internal and external finance are substitutable, notice that an entrepreneur can finance its assets using either internal funds ($B$) or external funds ($D$). In an economy without financing frictions impatient entrepreneurs finance most of their capital using external funds. In contrast, in an economy without external finance, entrepreneurs rely only on internal finance. Since internal finance is costlier than borrowing (due to impatience), a decline in the external finance to output ratio would lead to a reduction in the capital to output ratio. This decline is greater, the more impatient entrepreneurs are. As $\beta$ goes to 0, the capital output ratio in an economy without external finance would go to 0 as well.

Figure 5 shows a scatterplot of the capital to output and external finance to output ratio across countries in the data\(^\text{17}\). Consistent with the predictions of the theory, a lower external finance to GDP ratio is associated with a decline in the capital to output ratio. Countries with an external finance to GDP ratio of about 2.5, as in the US, also have a capital to output ratio of about 2.5. In contrast, countries with no external finance have an average $K/Y$ ratio of 1.4. A regression line through this data shows that the elasticity of $K/Y$ to $D/Y$ is equal to 0.51.

The Figure also reports the predictions of the model. Notice that the benchmark model we have studied, with $\beta = 0.92$, somewhat overpredicts the capital to output ratio in

\(^{17}\text{Recall that we use the capital and output data from Caselli (2005) and the finance data from Beck et. al (2000).}\)
the data. Moreover, it predicts that capital increases a bit more slowly with external finance than it does in the data: the elasticity of \( K/Y \) to \( D/Y \) is equal to 0.36. In contrast, the model with somewhat more impatient agents, with \( \beta = 0.85 \), does a very good job of reproducing the pattern in the data. The capital to output ratio is a bit lower than it is in the data, and the stock of capital is a bit more sensitive to external finance than it is in the data (the elasticity is equal to 0.56), but overall the model matches the data very well. In fact, when we calibrate \( \beta \) to match the relationship between the capital-output and finance ratio in the data, we find that a value of \( \beta = 0.86 \) fits the data best.

Since we have shown above that our model’s predictions regarding the size of TFP losses are not much greater in the economy with \( \beta = 0.85 \) (recall that the TFP gap between the US and an economy without external finance was equal to 7%), we conclude that our finding of small TFP losses do not reflect an overstatement of the ability of entrepreneurs to save internally. Had we assumed greater impediments to internal accumulation, the model would have produced much smaller capital-to-output ratios in poor countries than what we observe in the data.

5. Economy with Exit and Entry

We next ask: does allowing entry and exit change our earlier conclusions that the TFP losses from financing frictions are small? Do finance frictions distort the entry/exit margin by preventing productive agents from becoming entrepreneurs? Clearly, if permanent productivity shocks account for most of the unconditional dispersion in productivity, the entry/exit margin can only be distorted if some of the very productive entrepreneurs are relatively poor. But this will not be the case in the ergodic steady-state unless some of these entrepreneurs are young, so that they haven’t yet had a chance to accumulate assets and grow out of their borrowing constraint. Hence, in addition to allowing for a choice of entering/exiting entrepreneurship, we also assume that some agents die and are replaced each period by (relatively poor) newborn agents. We assume thus a constant hazard of death each period, \( 1 - p \). We show below that death is necessary in order to allow the model to account for the fact that some very large plants exit any given period in the data.

A. Environment

As earlier, we assume a continuum of agents of measure 1, indexed by \( i \). Each period the agent decides whether to be an entrepreneur or worker. Switching occupations entails no
cost and so these decisions are reversible each period.

A worker supplies 1 unit of labor inelastically at a wage rate $W$. As earlier, entrepreneurs have access to a technology that produces output using inputs of capital and labor:

$$Y_{it} = A_{it}^{1-\eta} \left( L_{it}^{\alpha} K_{it}^{1-\alpha} \right)^{\eta},$$

where $A_{it}$ is the agent’s productivity as an entrepreneur. We assume

$$\log(A_{it}) = a_{it} = Z_i + \tilde{a}_{it},$$

where, as earlier, $Z_i$ is a permanent productivity component, drawn from a Pareto distribution, and $\tilde{a}_{it}$ is an AR(1) process, as described earlier.

Both types of agents can save using a one period risk-free security. In addition, entrepreneurs can borrow within a period in order to finance labor and capital expenditure, but their ability to borrow is limited by the collateral constraint:

$$WL + K \leq \lambda B.$$

As earlier, we can compute the profits an agent can earn as an entrepreneur:

$$\pi(B, A) = \max_{K,L} AF(K, L) - (1 + r)WL - (r + \delta) K$$

subject to $WL + K \leq \lambda B$

We can write the agent’s value recursively as:

$$V(B, a) = \max_{B' \geq 0} \frac{C^{1-\gamma}}{1 - \gamma} + \beta p \int V(B', a') \pi(a'|a) da'$$

where $C = (1 + r) B + \max[\pi(B, A), W] - B'$,

and recall that $p$ is the constant survival probability.
Each period a measure \((1 - p)\) of agents are born. At birth agents draw a permanent productivity component, \(Z_i\), from a Pareto distribution characterized by \((\mu, H)\) and a variable component \(\tilde{a}_i = 0\). Moreover, they receive an endowment \(B_0(Z_i)\), deposited in an account with the financial intermediary. We assume that the endowment is potentially a function of the agent’s productivity as an entrepreneur. [One could interpret this dependence, as Evans and Jovanovic (1989) do, as reflecting the savings decisions of high-ability people who expected to become entrepreneurs one day. Another interpretation would be seed funding by a venture capitalist who finances the higher-ability would-be entrepreneur]. The newly born agent then chooses its occupation and faces the same problem as an old agent.

We assume, as earlier, that this is a small open economy so that agents can borrow at a risk-free rate \(r\). Let \(\mu(B,A)\) denote the ergodic measure of agents over asset holdings and productivity. Let \(I(B,A) = W > \pi(B,A)\) denote the choice of becoming a worker. Let \(L(B,A)\) denote the amount of labor demanded by an entrepreneur of type \((B,A)\). The equilibrium wage rate satisfies:

\[
\int I(B,A) \, d\mu(B,A) = \int L(B,A) \, (1 - I(B,A)) \, d\mu(B,A)
\]

**B. Parametrization**

In Table 7 we present the moments in the data that we would like our model to account for. These are now computed for the entire sample of plants, including those that are in sample for only a few years. The sample of plants now considerably increases, from 32,000 earlier, to 161,000 for Korea. Since we have shown earlier that the moments for Colombia are similar to those in Korea, we only report the moments for Korea in the Table and discuss below how the Colombian numbers compare.

We report the same set of moments that characterize the distribution of growth rates, persistence, size distribution of plants as earlier. A comparison of the first columns of Table 7 and Table 1 reveals that these moments are very similar for the larger unbalanced panel of plants we consider now.

In addition, we would like our model to account for the age-distribution and exit hazards of plants in the data. Notice in Panel D. of Table 7 that most plants are young (ages 1-5): 51%, with the rest of the sample roughly split between ages 6-10 and 10+. Also notice
that there is considerable amount of turnover in the data: the unconditional exit hazard is 1/3, mostly reflecting exit by very small plants. Larger plants, however, exit too. One way to see this is to compute the share of output accounted for by exiting plants. This is equal to 7% in the data, thus suggesting that some very large plants exit as well.

**Economy with no initial endowment**

Consider first an economy in which newly born agents enter with no endowment other than their labor income, $B_0(Z_i) = W$. We calibrate this economy using a similar procedure as described earlier: now the set of parameters also includes $p$, the survival probability, and we target the additional set of moments that describe the exit hazards in Panel D of Table 7. Since we now allow unproductive entrepreneurs to exit, our original setup has difficulty accounting for the size distribution of establishments. We therefore modify slightly the distribution of the permanent component of productivity, $Z_i$. In particular, we assume that a fraction $f_H$ of entrepreneurs have a permanent productivity component equal to $\bar{H}$, while the rest draw their productivity from the bounded Pareto with parameters $\mu, H$. We assume $\bar{H} \geq H$. We found that allowing a mass point in the upper tail of the distribution is necessary for the model to fit the size distribution of plants well.

Panel I of Table 7 shows that the model matches the moments in the data reasonably well. As in the data, most plants are young (64% in the model, 51% in the data), exit hazards are large (30% of plants exit in the model, 33% in the data), and exiting plants account for a substantial share of output (7% in the model and in the data). Table 8 reports the parameter values that achieve this fit.

Panel A of Table 9 reports some of the key predictions of this variation of the model. A lot more establishments are now constrained than in the economy without exit and entry. The medium external finance premium is 10% and is much more dispersed. For example, the 90th percentile is equal to 38% and the 99th percentile is equal to 70%. This dispersion in the internal cost of funds manifests itself in much greater TFP losses. These are equal to 15.1% for our economy, that, recall, is calibrated to the 1.2 debt-to-GDP ratio in Korea. The TFP losses are thus almost 4 times greater than in the economy without exit and entry (recall, equal to 3.9%). Interestingly, most of these losses reflect misallocation of factors among existing plants, not distortions along the entry-exit margin. To see this, we decompose the TFP losses into those arising due to an inefficient allocation of agents into entrepreneurship. These latter losses are much smaller, 0.2%, reflecting that most marginal entrepreneurs are
small and account for a small share in aggregate output.

The model also predicts much greater cross-country TFP differences when varying the level of financial development. The TFP losses in an economy calibrated to the US 2.3 debt-to-GDP ratio are equal to 5.6%, while those in an economy with no external finance are equal to 22.6%, thus a 17% gap, much greater than the 5% we reported earlier.

The reason TFP losses are much greater here is that newly born agents that have high ability, $Z_i$, enter entrepreneurship almost immediately in this version of the model, despite the fact that initially they have little assets and are thus severely constrained. Profits from operating a plant are much greater for highly talented entrepreneurs, in equilibrium, than the relatively low wage they can earn as a workers. Since such entrepreneurs are, initially, very poor, they cannot afford the efficient amount of capital and labor and this reflects in relatively high TFP losses.

We note, however, that this version of the model is at odds with the dynamics of plants in the data. To see this, Figure 6 shows the relationship between growth rates and age in the model (dashed line) and compares it to the data (dots). Notice that the youngest establishments grow a lot quicker than in the data (for example, the average growth rate for a 2-year old plant is 25% in the model and only 10% in the data). Panel F of Table 7 shows that young (ages 1-5) plants grow 15% faster in the model than older (ages 10+) plants do. In contrast, they grow only 5% faster in the data. Similarly, plants aged 6-10 grow 8% faster in the model and only 2% faster in the data. Establishments grow much faster in the model because of borrowing constraints: as establishments age, they accumulate internal funds and grow because of the ability to hire capital and labor\(^{18}\).

Another way to see that young establishments are too constrained in the model is to compare the returns to capital (recall these are a function of shadow cost of funds) for establishment of different age groups. We compute returns to capital as the average product of capital and find that the model predicts that these returns are much greater for the youngest plants (ages 1-5 and 6-10) than they are in the data. Panel F of Table 7 shows that the average product of capital is 25% greater in the model for plants aged 1-5 than for plants that are 10 years old or older; the corresponding statistic is equal to 4% in the data. Similarly, plants aged 6-10 have a much greater average product of capital in the model (24% higher) than in the data (6% higher). Once again, this second measure suggests young entrepreneurs

\(^{18}\)This is a typical predictions of this class of models. See for example Cooley and Quadrini (2001).
are much too constrained in our model than in the data.\(^{19}\)

Since growth rates and the average product of capital are, in our theory, strongly tied to the extent to which establishments are constrained, we conclude that this version of the model generates TFP losses for the wrong reason, by implying that young establishments are much more constrained than what they are in the data.

**Economy with endowment**

The counterfactual predictions above can be easily addressed by assuming that newly entering agents receive an endowment that depends on their ability \(Z_i\). Let

\[
B_0(Z_i) = W + \phi (WL(Z_i) + K(Z_i))
\]

where \(L(Z_i)\) and \(K(Z_i)\) are the efficient amount of labor an entrepreneur would hire absent financing frictions. Here, \(\phi\) governs the extent to which a new agent’s endowment is correlated with its ability. In the limit, if \(\phi = 1\), entering establishments can achieve the efficient scale without borrowing externally. We assume \(\phi \in (0, 1)\) and calibrated this parameter, together with the others, by requiring that the model matches the statistics in Panel F of Table 7 on the characteristics of young versus old plants, in addition to the other moments we have targeted above.

It turns out that a value of \(\phi = 0.35\) best fits this feature of the data. Figure 6 shows that now the model fits the growth-age relationship very well: as in the data, newly entering plants grow about 10% faster. Panel F of Table 7 shows that the model also fits well the relationship between the returns to capital and age, though it implies that the youngest plants are somewhat more constrained (a 9% higher average product of capital than 10+ plants) than in the data (4% higher average product of capital).

Consider next the implications for TFP in this setup. Since entering establishments are now less constrained, the model now produces much smaller dispersion in returns to factors. Although the majority of plants (75%) are still constrained in our Korean calibration, the interquartile range of the shadow cost of funds declines to 0.04 (0.09 in the economy with no endowment). Similarly the 99th percentile of \(\tilde{r}\) declines significantly, from 0.70 to 0.21.

\(^{19}\)We have computed similar statistics for establishments in Colombia and found similar numbers. Plants aged 1-5 grow only 11% faster than those older than 10 years, while plants aged 6-10 grow only 2% faster. As for the average product of capital, in Colombia it, in fact, increases with age.
Entering entrepreneurs that are extremely productive are no longer as constrained as earlier.

The decrease in the dispersion of the shadow cost of funds implies now that aggregate productivity losses are much smaller. The TFP losses are 1.7% for the US calibration, 4.7% in Korea, and 6.6% for an economy without external funds. As in the economy without exit and entry, finance frictions generate a fairly small TFP gap, about 5%, between US and economies with no external finance.

**Economy with fixed costs**

In the previous two examples the source of non-convexity in an agent’s decision rule was the occupational choice decision. The amount of entry and exit was pinned down by the outside option of the entrepreneur, its wage earnings. An alternative way to generate establishment turnover is to assume that entrepreneurs must pay a fixed cost in order to operate the technology. We next study an economy with such a feature. One hypothesis is that requiring entrepreneurs to pay a fixed cost would reduce their ability to accumulate internal funds and grow out of the borrowing constraint. The conjecture is thus that the TFP implications of an economy with fixed costs are different from those of the setup we have studied earlier.

To explore this conjecture, we next modify the assumptions on the production technology and assume that operating the plant requires a fixed (overhead) cost, denominated in units of labor. Assume that

\[
Y_{it} = \begin{cases} 
A_{it}^{1-\eta} ((L_{it} - f)^\alpha K_{it}^{1-\alpha})^\eta & \text{if } L_{it} \geq f \\
0, & \text{otherwise}
\end{cases}
\]

where \( f \) is the fixed operating cost. We also assume that entrepreneurs no longer have the option to switch occupations and become workers. Rather, they have the option to shut down in any particular period and earn 0 profits. Finally, we assume a constant measure of workers that each supplies labor inelastically.

We follow Bartelsman, Haltiwanger and Scarpetta (2008) and calibrate the size of the fixed cost, together with the other parameters of the model, to match the amount of plant turnover, as well as the fraction of young and old plants. This is the same set of moments we have targeted earlier. Table 7 shows that the model matches the data quite well, although it

34
once again overstates the returns to capital of the youngest plants. Table 8 shows that the fixed cost required to match the moments in the data amounts to 9% of the overall labor bill in the aggregate.

As for the size of the TFP losses, Table 9 shows that these are similar to those we had in the economy without a fixed cost. Once again, allowing the model to match the micro features of the data implies that the TFP losses it predicts are quite small: the gap between the TFP in the U.S. calibration and that in an economy without external finance is only 5.3%. We thus conclude that the exact source of non-convexity is not important for our results. Even though entering entrepreneurs pay a large fraction of their dividends to cover their fixed costs, their returns to savings are very high and so they find it optimal to postpone consumption and quickly grow out of their borrowing constraint.

6. Conclusions

We study a model of establishment dynamics with borrowing constraints. The model, when parameterized to account for the salient features of the plant-level data, predicts that even extreme financing frictions produce modest (5-7%) TFP losses from misallocation. These TFP losses are much smaller than the 60% TFP gap between countries with little external finance and the U.S.

We emphasize that ours is not an impossibility result: we do not argue that financing frictions cannot generate large aggregate efficiency losses. Indeed, we present parameterizations of the model that predict TFP losses as large as 30%. Such parameterizations, however, miss important features of the micro data.

These results reflect our focus on a very specific mechanism. We asked: to what extent do finance frictions distort the optimal reallocation of factors of production across entrepreneurs that differ in their productivity. We thus do not interpret our findings as evidence against an important link between finance and TFP. To the extent to which finance frictions distort the adoption of newer and better technologies, or distort the process of learning-by-doing, as in the work of Parente (1994), their effect on TFP is potentially much greater. An extension of our analysis along these lines remains an exciting topic for future research.

Our message is that financing frictions cannot endogenously generate much dispersion in the marginal product of capital. Finance, however, may still play an important role in distorting allocations if entrepreneurs differ in the interest rates at which they can borrow. If
some entrepreneurs have access to cheap credit (as in the case of business conglomerates or due to subsidies from state-owned banks) and others do not, the TFP losses from misallocation can be quite large, as pointed out by Restuccia-Rogerson (2008) and Hsieh and Klenow (2009). Whether such differences in terms of financing can indeed account for the large cross-country TFP differences is an open empirical question.

References


Table 1: Establishment-level facts

<table>
<thead>
<tr>
<th>A. Distribution of growth rates</th>
<th>Korea</th>
<th>Colombia</th>
<th>Korea (10 + plants)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta y_{it})$</td>
<td>0.54</td>
<td>0.49</td>
<td>0.53</td>
</tr>
<tr>
<td>$\text{kurt}(\Delta y_{it})$</td>
<td>12.9</td>
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<td>13.00</td>
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<tr>
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<td>0.36</td>
<td>0.47</td>
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<table>
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<tr>
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<tr>
<td>$\rho(y_{it}, y_{t+1})$</td>
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<td>0.96</td>
<td>0.92</td>
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<td>0.85</td>
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</table>

<table>
<thead>
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<th>C. Size distribution</th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>fraction of $Y$ largest 1%</td>
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<td>0.30</td>
<td>0.49</td>
</tr>
<tr>
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<td>fraction of $Y$ largest 10%</td>
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<thead>
<tr>
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<td></td>
<td>1.2</td>
<td>0.3</td>
<td>1.2</td>
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| # plants                        | 31543 | 4787     | 26833              |

Note: $y$ is the log of value added (revenue net of spending on intermediate inputs)
Table 2: Parameter values

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<tr>
<td>EIS</td>
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<td>discount factor</td>
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Table 3: Moments in Model and Data

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<td><strong>A. Distribution of growth rates</strong></td>
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<td>$\sigma(\Delta y_{it})$</td>
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<td>0.51</td>
<td>0.51</td>
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<tr>
<td><strong>C. Size distribution</strong></td>
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<td>fraction of $Y$ largest 1%</td>
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<td>fraction of $Y$ largest 5%</td>
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<td>1.2</td>
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<tr>
<td>RMSE, %</td>
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Note: $y$ is the log of value added (revenue net of spending on intermediate inputs)
Table 4: Model predictions

<table>
<thead>
<tr>
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<th>Colombia</th>
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<tr>
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A. Size of financial frictions

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<td>0.05</td>
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B. Micro-moments

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<tr>
<th></th>
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C. TFP losses from misallocation, %

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Table 5: Counterfactual experiments. No permanent component.

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<tr>
<th></th>
<th>Data</th>
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<th>II. Low persistence. Match size distribution</th>
<th>III. Low persistence. Shocks as in I</th>
</tr>
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<td><strong>A. Distribution of growth rates</strong></td>
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<td><strong>B. Persistence</strong></td>
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<tr>
<td>$\rho(y_{it}, y_{it-1})$</td>
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<td>0.93</td>
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<td>$\rho(y_{it}, y_{it-3})$</td>
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<tr>
<td>$\rho(y_{it}, y_{it-5})$</td>
<td>0.86</td>
<td>0.69</td>
<td>0.34</td>
<td>0.30</td>
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<tr>
<td><strong>C. Size distribution</strong></td>
<td></td>
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<tr>
<td>fraction of $Y$ largest 1%</td>
<td>0.57</td>
<td>0.52</td>
<td>0.44</td>
<td>0.13</td>
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<tr>
<td>fraction of $Y$ largest 10%</td>
<td>0.84</td>
<td>0.88</td>
<td>0.88</td>
<td>0.48</td>
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<tr>
<td><strong>D. TFP losses, %</strong></td>
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<tr>
<td>Korea (1.2)</td>
<td></td>
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</tr>
<tr>
<td>Colombia (0.3)</td>
<td></td>
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<tr>
<td>Worst-case</td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>10.5</td>
<td>18.3</td>
<td>6.6</td>
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<td></td>
<td>18.1</td>
<td>29.5</td>
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<td></td>
<td>54.3</td>
<td>69.9</td>
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<td><strong>Parameters</strong></td>
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<tr>
<td>persistence shocks</td>
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<tr>
<td>$\rho$</td>
<td>0.92</td>
<td>0.80</td>
<td>0.80</td>
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<tr>
<td>stand. dev. shocks</td>
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<tr>
<td>$\sigma_{\epsilon}$</td>
<td>1.19</td>
<td>2.27</td>
<td>1.19</td>
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Table 6: Sensitivity analysis

<table>
<thead>
<tr>
<th>Parameters</th>
<th>I. Benchmark</th>
<th>II. $\beta = 0.85$ (more impatience)</th>
<th>III. $\theta = 0.5$ (K &amp; L less substit.)</th>
<th>IV. $\eta = 0.95$ (greater span of control)</th>
</tr>
</thead>
<tbody>
<tr>
<td>persistence shocks</td>
<td></td>
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<td>$\rho$</td>
<td>0.74</td>
<td>0.73</td>
<td>0.78</td>
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<td>stand. dev. shocks 1</td>
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<tr>
<td>$\sigma_{1x}$</td>
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<td>0.92</td>
<td>0.71</td>
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<td>$\sigma_{2x}$</td>
<td>1.78</td>
<td>1.41</td>
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<td>frequency shocks 2</td>
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<td>$\kappa$</td>
<td>0.070</td>
<td>0.072</td>
<td>0.049</td>
<td>0.022</td>
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<td>Pareto shape</td>
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<tr>
<td>$\mu$</td>
<td>3.64</td>
<td>3.90</td>
<td>2.93</td>
<td>12.64</td>
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<tr>
<td>Pareto upper bound</td>
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<td>$H$</td>
<td>9.02</td>
<td>9.00</td>
<td>8.47</td>
<td>9.63</td>
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<td>TFP losses, %</td>
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<tr>
<td>US (2.3)</td>
<td>1.0</td>
<td>2.2</td>
<td>0.7</td>
<td>2.9</td>
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<tr>
<td>Korea (1.2)</td>
<td>3.9</td>
<td>6.5</td>
<td>3.2</td>
<td>5.6</td>
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<tr>
<td>Colombia (0.3)</td>
<td>5.4</td>
<td>8.8</td>
<td>5.4</td>
<td>6.9</td>
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<tr>
<td>Worst-case</td>
<td>8.6</td>
<td>12.6</td>
<td>8.3</td>
<td>8.4</td>
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Table 7: Moments in Economy with Exit/Entry

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<tr>
<th></th>
<th>Data (Korea)</th>
<th>I. No endowment</th>
<th>II. Endowment</th>
<th>III. Fixed cost</th>
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<tbody>
<tr>
<td>A. Distribution of growth rates</td>
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<tr>
<td>$\sigma(\Delta y_{it})$</td>
<td>0.56</td>
<td>0.56</td>
<td>0.55</td>
<td>0.56</td>
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<tr>
<td>$kurt(\Delta y_{it})$</td>
<td>11.4</td>
<td>8.3</td>
<td>6.5</td>
<td>6.1</td>
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<tr>
<td>$iqr(\Delta y_{it})$</td>
<td>0.51</td>
<td>0.49</td>
<td>0.43</td>
<td>0.46</td>
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<tr>
<td>B. Persistence</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$\rho(y_{it}, y_{i,t-1})$</td>
<td>0.92</td>
<td>0.94</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td>$\rho(y_{it}, y_{i,t-3})$</td>
<td>0.88</td>
<td>0.87</td>
<td>0.84</td>
<td>0.86</td>
</tr>
<tr>
<td>$\rho(y_{it}, y_{i,t-5})$</td>
<td>0.86</td>
<td>0.82</td>
<td>0.80</td>
<td>0.83</td>
</tr>
<tr>
<td>C. Size distribution</td>
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<td></td>
</tr>
<tr>
<td>fraction of $Y$ by largest 1%</td>
<td>0.53</td>
<td>0.45</td>
<td>0.46</td>
<td>0.39</td>
</tr>
<tr>
<td>fraction of $Y$ by largest 5%</td>
<td>0.72</td>
<td>0.82</td>
<td>0.66</td>
<td>0.83</td>
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<tr>
<td>fraction of $Y$ by largest 10%</td>
<td>0.79</td>
<td>0.89</td>
<td>0.78</td>
<td>0.87</td>
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<tr>
<td>fraction of $Y$ by largest 20%</td>
<td>0.87</td>
<td>0.94</td>
<td>0.88</td>
<td>0.92</td>
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<tr>
<td>D. Age and exit hazards</td>
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<tr>
<td>fraction age = 1 - 5</td>
<td>0.51</td>
<td>0.64</td>
<td>0.62</td>
<td>0.61</td>
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<tr>
<td>fraction age = 6 - 10</td>
<td>0.26</td>
<td>0.14</td>
<td>0.17</td>
<td>0.19</td>
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<tr>
<td>fraction age &gt; 10</td>
<td>0.23</td>
<td>0.22</td>
<td>0.21</td>
<td>0.20</td>
</tr>
<tr>
<td>exit hazard</td>
<td>0.33</td>
<td>0.30</td>
<td>0.25</td>
<td>0.22</td>
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<tr>
<td>output share exiting plants</td>
<td>0.07</td>
<td>0.07</td>
<td>0.08</td>
<td>0.08</td>
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<tr>
<td>E. Debt-to-GDP</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
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<tr>
<td>F. Young plants</td>
<td></td>
<td></td>
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<tr>
<td>mean $\Delta y$ if age = 1 - 5</td>
<td>0.05</td>
<td>0.15</td>
<td>0.06</td>
<td>0.05</td>
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<tr>
<td>mean $\Delta y$ if age = 6 - 10</td>
<td>0.02</td>
<td>0.08</td>
<td>0</td>
<td>0</td>
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<tr>
<td>$\Delta Y/K$ if age = 1 - 5</td>
<td>0.04</td>
<td>0.25</td>
<td>0.09</td>
<td>0.17</td>
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<tr>
<td>$\Delta Y/K$ if age = 6 - 10</td>
<td>0.06</td>
<td>0.24</td>
<td>0.06</td>
<td>0.08</td>
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</tbody>
</table>
### Table 8: Parameters in Economy with Exit/Entry

<table>
<thead>
<tr>
<th></th>
<th>I. No endowment</th>
<th>II. Endowment</th>
<th>III. Fixed cost</th>
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</thead>
<tbody>
<tr>
<td>collateral constraint</td>
<td>$\lambda$</td>
<td>2.3</td>
<td>2.1</td>
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<tr>
<td>persistence shocks</td>
<td>$\rho$</td>
<td>0.35</td>
<td>0.55</td>
</tr>
<tr>
<td>stand. dev. shocks</td>
<td>$\sigma_{,x}$</td>
<td>1.30</td>
<td>1.08</td>
</tr>
<tr>
<td>Pareto shape</td>
<td>$\mu$</td>
<td>2.5</td>
<td>2.3</td>
</tr>
<tr>
<td>Pareto upper bound</td>
<td>ln$H$</td>
<td>17.0</td>
<td>15.3</td>
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<tr>
<td>mass point</td>
<td>ln$H_{bar}$</td>
<td>17.0</td>
<td>17.5</td>
</tr>
<tr>
<td>mass at upper bound,%</td>
<td>$f_{H}$</td>
<td>-</td>
<td>0.01</td>
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<tr>
<td>probability survival</td>
<td>$p$</td>
<td>0.95</td>
<td>0.94</td>
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<tr>
<td>endowment</td>
<td>$\phi$</td>
<td>-</td>
<td>0.35</td>
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<tr>
<td>fixed cost</td>
<td>$f$</td>
<td>-</td>
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</table>
Table 9: Predictions of Economy with Exit/Entry

<table>
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<tr>
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<th>I. No endowment</th>
<th>II. Endowment</th>
<th>III. Fixed cost</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Size of financial frictions</strong></td>
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<tr>
<td>Fraction constrained</td>
<td>0.85</td>
<td>0.75</td>
<td>0.72</td>
</tr>
<tr>
<td>Median premium if constrained</td>
<td>0.10</td>
<td>0.07</td>
<td>0.08</td>
</tr>
<tr>
<td>IQR premium if constrained</td>
<td>0.09</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>90% premium if constrained</td>
<td>0.38</td>
<td>0.13</td>
<td>0.16</td>
</tr>
<tr>
<td>99% premium if constraint</td>
<td>0.70</td>
<td>0.21</td>
<td>0.25</td>
</tr>
<tr>
<td><strong>B. TFP losses, %</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US (D/Y = 2.3)</td>
<td>5.6</td>
<td>1.7</td>
<td>1.9</td>
</tr>
<tr>
<td>Korea (D/Y = 1.2)</td>
<td>15.1</td>
<td>4.7</td>
<td>5.3</td>
</tr>
<tr>
<td>Colombia (D/Y = 0.3)</td>
<td>20.7</td>
<td>6.3</td>
<td>6.9</td>
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<tr>
<td>No Debt</td>
<td>22.6</td>
<td>6.6</td>
<td>7.2</td>
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</table>
Figure 1: TFP vs. External Finance

External Finance to GDP
Figure 2: Decision rules

A. Shadow cost of funds

B. Savings, $b'/b$

Frictionless

With borrowing constraint
Figure 3: Impulse response to a productivity shock

A. Productivity, $a$

B. Shadow cost of funds, $r$

C. Capital Stock, $K$, log

D. Assets, $B$, log

- With borrowing constraint
- Frictionless
Figure 4: Productivity vs. shadow cost of funds

- Shadow cost of funds, $\tilde{r}$
- Productivity, $a$
Figure 5: K/Y vs. External Finance

\[ \beta = 0.92 \text{ (elast. = 0.36)} \]

\[ \beta = 0.85 \text{ (elast. = 0.56)} \]

Data (elast. = 0.51)
Figure 6: Growth rates vs. age

- Model with no startup funds
- Model with startup funds
- Korean data